Assignment

1. The graph drawn with the thick line is a horizontal shift, a vertical shift, or both a horizontal shift and a vertical shift of the original graph drawn with the thin line.

In each case,

i) estimate the amount of the shift and the direction of the shift

ii) sketch the midline of the new graph and state its equation.

a) 

i) horizontal shift 2 units right

ii) $y = 0$

b) 

i) vertical shift 2 units up

ii) $y = 2$

c) 

i) horizontal shift 1.5 units right and vertical shift 2 units up

ii) $y = 2$
2. In each diagram, the sinusoidal function \( y = \sin x \) has been graphed with a thin line. Determine if the sinusoidal function graphed with the thick line differs from the graph of \( y = \sin x \) by

- a change in amplitude
- a horizontal phase shift
- a change in period
- a vertical shift in the midline.

There may be more than one change involved.
3. Complete the following table. Record the amplitude, the period (as an exact value), and the midline value in the first three rows. In the last row, answer “yes” if there is a phase shift from the graph of $y = \sin x$ and “no” if not.

<table>
<thead>
<tr>
<th>a) $y = \sin x$</th>
<th>b) $y = 0.1 \sin(4\pi x) - 5$</th>
<th>c) $y = 4 \sin \left(\frac{\pi}{2} x + 1\right)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>amplitude</td>
<td>1</td>
<td>amplitude</td>
</tr>
<tr>
<td>period</td>
<td>$\frac{\pi}{4}$</td>
<td>0.5</td>
</tr>
<tr>
<td>midline value</td>
<td>0</td>
<td>midline value</td>
</tr>
<tr>
<td>phase shift?</td>
<td>no</td>
<td>phase shift?</td>
</tr>
</tbody>
</table>

4. Complete the following table. Record the amplitude, the period (to the nearest hundredth), and the midline value in the first three rows. In the last row, answer “yes” if there is a phase shift from the graph of $y = \sin x$ and “no” if not.

<table>
<thead>
<tr>
<th>a) $y = 9 \sin(1.57x + 15)$</th>
<th>b) $y = 9 \sin 1.57x + 15$</th>
<th>c) $y = 2 \sin(0.3x + 7) - 9$</th>
</tr>
</thead>
<tbody>
<tr>
<td>amplitude</td>
<td>9</td>
<td>amplitude</td>
</tr>
<tr>
<td>period</td>
<td>$4.00$</td>
<td>period</td>
</tr>
<tr>
<td>midline value</td>
<td>0</td>
<td>midline value</td>
</tr>
<tr>
<td>phase shift?</td>
<td>yes</td>
<td>phase shift?</td>
</tr>
</tbody>
</table>

5. Consider sinusoidal functions of the form $y = a \sin(bt + c) + d$. Write the equation of a sinusoidal function with no phase shift with the following characteristics.

a) amplitude 7 units, period $2\pi$ radians, and midline equation $y = -4$

$$a = 7, \quad b = \frac{2\pi}{2\pi} = 1, \quad y = 7 \sin t - 4$$

b) amplitude 2.5 units, period 1 radian, and a midline value of 6

$$a = 2.5, \quad b = \frac{2\pi}{1} = 2\pi, \quad y = 2.5 \sin(2\pi t) + 6$$

Copyright © by Absolute Value Publications. This book is NOT covered by the Canopy agreement.
6. The graph represents a sinusoidal function of the form \( y = a \sin(bx + \pi) + d \), with \( a > 0 \). The maximum and the minimum values are integers.

a) Determine the amplitude, the period, and the midline value.

\[
\text{amplitude } = \frac{2 - (-4)}{2} = 3 \quad \text{period } = 4 \times \frac{\pi}{4} = \pi \\
\text{midline value } = \frac{2 + (-4)}{2} = -1
\]

b) Determine values for \( a, b, \) and \( d \) and write the equation of the function.

\[
a = 3 \quad b = \frac{2\pi}{\pi} = 2 \quad d = -1 \quad y = 3 \sin(2x + \pi) - 1
\]

c) Determine, to the nearest tenth, the function value when \( x = -\frac{\pi}{3} \).

\[
y = 3 \sin\left(2\left(-\frac{\pi}{3}\right) + \pi\right) - 1 = 1.598... \approx 1.6
\]

7. The graph represents a sinusoidal function of the form \( y = a \sin\left(bx + \frac{\pi}{4}\right) + d \), with \( a > 0 \).

The maximum and minimum values are integers.

Determine the equation of the graph.

\[
\text{amplitude } = \frac{7-1}{2} = 3 \quad a = 3 \\
\text{period } = 8 \times \frac{\pi}{2} = 4\pi \\
\text{midline value } = \frac{7+1}{2} = 4 \quad d = 4 \\
\]

\[
y = 3 \sin\left(\frac{1}{2}x + \frac{\pi}{4}\right) + 4
\]

8. The sine graph shows a maximum value of 20, a minimum value of 10, and passes through the point (12, 15).

Determine the equation of the graph in the form \( y = a \sin bt + d \).

\[
\text{amplitude } = \frac{20-10}{2} = 5 \quad a = 5 \\
\text{period } = 2 \times 12 = 24 \\
\text{midline value } = \frac{20+10}{2} = 15 \quad d = 15
\]

\[
y = 5 \sin\left(\frac{\pi}{12} t\right) + 15
\]

Copyright © by Absolute Value Publications. This book is NOT covered by the Canopy agreement.
9. The range of the graph of the sinusoidal function shown is \( \{ y \mid -3 \leq y \leq 9, y \in \mathbb{R} \} \).

   The graph has an equation in the form
   \[
y = a \sin \left( b x + \frac{\pi}{8} \right) + d, \quad a > 0.
   \]

   a) Determine the equation of the midline.
   \[
y = \frac{3 + (-3)}{2} = 3
   \]
   b) Determine the equation of the graph.
   \[
   \begin{align*}
   \text{amplitude} &= \frac{3 - (-3)}{2} = 6, \quad a = 6 \\
   \text{period} &= 16 \times \frac{\pi}{4} = 4\pi, \quad b = \frac{4\pi}{4\pi} = \frac{1}{2} \\
   d &= 3
   \end{align*}
   \]
   \[
y = 6 \sin \left( \frac{1}{2} x + \frac{\pi}{8} \right) + 3
   \]

10. Consider the graph of the sinusoidal function \( f(t) = -2 \sin (4t - \pi) - 3 \).

   a) State the amplitude and the midline value.
   \[
   \begin{align*}
   \text{amplitude} &= 2, \quad \text{midline value} = -3
   \end{align*}
   \]

   b) Use the results in a) to determine the maximum value, the minimum value, and the range of the graph.
   \[
   \begin{align*}
   \text{midline value} + \text{amplitude} &= -1 \\
   \text{midline value} - \text{amplitude} &= -5
   \end{align*}
   \]
   \[
   \text{range} = \{ f(t) \mid -5 \leq f(t) \leq -1, \quad f(t) \in \mathbb{R} \}
   \]

11. Determine the range of the functions represented below.

   a) \( y = 2 \sin x - 2 \) \quad b) \( y = a \sin(bt + c) + d \), where \( a > 0 \)

   \[
   \begin{align*}
   \text{amplitude} &= 2, \quad \text{midline} = -2 \\
   -2 + 2 &= 0, \quad -2 - 2 = -4
   \end{align*}
   \]

   \[
   \begin{align*}
   \text{max.} &= d + a, \quad \text{min.} = d - a \\
   \text{range} &= \{ y \mid -4 \leq y \leq 0, \quad y \in \mathbb{R} \}
   \end{align*}
   \]

12. Which of the following graphs has the same \( x \)-intercepts as the graph of \( y = \sin x \)?

   A. \( y = \sin 4x \)
   B. \( y = 4 \sin x \) \quad \text{change in amplitude only}
   C. \( y = \sin x + 4 \)
   D. \( y = \sin(x + 4) \)
13. The period, to the nearest tenth, of the function \( y = \sin 0.25x \), where \( x \) is in radians, is \[ 25.1 \]

(Record your answer in the numerical response box from left to right.)

Use the following information to answer the next question.

The graphs below represent sinusoidal functions of the form \( y = a \sin(bx + c) + d \).

14. Write the graph number in which \( c = 0 \) and \( d = 0 \) in the first box.

Write the graph number in which \( c \neq 0 \) and \( d = 0 \) in the second box.

Write the graph number in which \( c = 0 \) and \( d \neq 0 \) in the third box.

Write the graph number in which \( c \neq 0 \) and \( d \neq 0 \) in the last box.

(Record your answer in the numerical response box from left to right.)

\[
\begin{align*}
\text{c} &= 0 \quad \text{no phase shift, midline intersects y-axis} \\
\text{d} &= 0 \quad \text{no vertical shift, midline} = 0
\end{align*}
\]
Assignment

1. The first Ferris wheel ever built was created by a bridge builder by the name of George W. Ferris in 1893.
The diameter of the Ferris wheel was approximately 76 metres and the maximum height of the wheel was approximately 80 metres.
There were 36 wooden carts on the wheel, with each cart able to hold approximately 60 people.

<table>
<thead>
<tr>
<th>Time (t) minutes</th>
<th>Height (h) metres</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>4</td>
</tr>
<tr>
<td>2.25</td>
<td>42</td>
</tr>
<tr>
<td>4.5</td>
<td>80</td>
</tr>
<tr>
<td>6.75</td>
<td>42</td>
</tr>
<tr>
<td>9</td>
<td>4</td>
</tr>
</tbody>
</table>

The Ferris wheel was introduced to the world at the 1893 World’s Fair in Chicago.
The illustration shown below is a copy of a photograph of the original wheel.

a) If the wheel rotated every nine minutes, use the data in the table to sketch a sinusoidal graph that represents the height of a cart in metres, as a function of time in minutes. Assume that the cart is at its lowest point at \( t = 0 \), and draw one complete cycle.

![Graph of the Ferris wheel](image)

b) Use sinusoidal regression to determine the equation of the graph in the form \( h(t) = a \sin(bt + c) + d \). Round the parameters to the nearest hundredth if required.

\[ h(t) = 38 \sin(0.70t - 1.57) + 4.2 \]

c) How high, to the nearest metre, is the cart 5 minutes after the wheel starts rotating?

\[ t = 5 \quad h(t) = 77.70... \quad 78 \text{ metres} \]

d) How many seconds after the wheel starts rotating does the cart first reach 10 metres from the ground? Answer to the nearest second.

\[ \text{graph} \quad y_2 = 10 \quad \text{intersect at} \quad t = 0.8159... \min \quad 49 \text{ seconds} \]
Use the following information to answer the next question.

In Inverdeen harbour, the maximum depth of water is 22 metres, occurring at 1 a.m. and 1 p.m., as shown on the grid below.

The minimum depth of water is 6 metres, occurring at 7 a.m. and 7 p.m.

The depth is 14 metres at 4 a.m., 10 a.m., 4 p.m., and 10 p.m.

Assume that the relation between the depth of water, \( y \) metres, and the time, \( t \) hours, is a sinusoidal function.

2. a) On the graph below (in which \( t = 0 \) at midnight), mark the key points from the information above with dots. The maximum points are shown.
   Connect the points to show the sinusoidal function that represents the data.

   ![Graph of depth vs time]

   midnight  midday  midnight

b) State the amplitude, period, and midline value for the sinusoidal function.
   (Include units in the answers.)
   \[ \text{amplitude} = \frac{22 - 6}{2} = 8 \text{ metres} \quad \text{period} = 12 \text{ hours} \quad \text{midline value} = \frac{22 + 6}{2} = 14 \text{ metres} \]

c) Using the values in b), determine an equation of the sinusoidal function in the form \( y = a \sin\left(bt + \frac{\pi}{3}\right) + d \).
   \[ a = 8 \quad b = \frac{2\pi}{12} = \frac{\pi}{6} \quad y = 8 \sin\left(\frac{\pi}{6} t + \frac{\pi}{3}\right) + 14 \]

   \[ d = 14 \]

d) Calculate the depth of the water, to the nearest tenth of a metre, at midnight.
   \[ t = 0 \quad y = 8 \sin\left(\frac{\pi}{3}\right) + 14 = 20.928\ldots = 20.9 \text{ metres} \]
e) i) What is the value of \( t \) at 3:30 pm? \( \frac{15.5}{12 + 3.5} \)

ii) Calculate the depth of the water, to the nearest tenth of a metre, at 3:30 pm.

\[
y = 8 \sin \left( \left( \frac{\pi}{3} \times 15.5 \right) + \frac{\pi}{3} \right) + 14 = 15.07 \ldots = 15.1 \text{ metres}
\]

f) Perform a sinusoidal regression using the data points on the graph to determine a sinusoidal regression function in the form \( y = a \sin(bx + c) + d \).
Round each parameter to 2 decimal places if necessary.

\[
y = 8 \sin \left( 0.52t + 1.05 \right) + 14
\]

g) Use the sinusoidal regression function in f) to calculate, to the nearest tenth of a metre, the depth of the water at midnight and at 3:30 pm.

\[
\text{midnight: } 20.9 \text{ metres} \quad \text{3:30 pm: } 16.1 \text{ metres}
\]

3. For a college research project, a student used statistical sampling techniques to estimate the population of opiliones ("Daddy Long Legs") in a region of St. Albert over a 12 month period. A scatterplot of the data indicated to her that a sinusoidal function was appropriate as a model.

The graphing calculator screen shows the graph of the sinusoidal regression function determined from her data. The indicated points are points on the graph.

The point \((0, 3142)\) indicates a minimum population of 3142 at the beginning of January.

The point \((6, 9454)\) indicates a maximum population at the beginning of July, and the point \((12, 3142)\) represents the population at the end of the year.

a) Use the given data to determine the sinusoidal regression function she obtained. Answer in the form \( y = a \sin(bx + c) + d \), rounding the parameters to four decimal places if required.

\[
y = 3156 \sin \left( 0.5236 \times -1.5708 \right) + 6298
\]

b) Using the sinusoidal regression equation, determine the period of the graph. Explain from the context of the question what this value represents.

The population increase and decrease every 0.5236 months is a yearly cycle.

c) Determine, to the nearest 50, the number of Daddy Long Legs at the beginning of August.

\[
x = 7 \quad y = 9031.17... = 9050
\]

d) To the nearest tenth of a month, determine the number of months that the number of Daddy Long Legs was greater than 5000.

\[
\text{graph } y_2 = 5000 \text{ intersect at 2.19... and 9.80... 7.6 months}
\]

e) The student found that data from a neighbouring region showed figures approximately double those in St. Albert. If a sinusoidal regression was done with the data from the neighbouring region, how would the graph of that data compare with the above graph in terms of amplitude, period, and midline?

amplitude and midline would double

period would not change

---

Copyright © by Absolute Value Publications. This book is NOT covered by the Cancopy agreement.
4. The graph shows the height, $h$ metres above the ground, of a person in a chair on a Ferris wheel after $t$ seconds as the wheel completes two revolutions.

The minimum height of the Ferris wheel is 2 metres and the maximum height is 20 metres.

a) Determine the amplitude and equation of the midline of the graph.

\[
\text{amplitude } = \frac{20 - 2}{2} = 9 \text{ metres} \quad \text{midline } = \frac{20 + 2}{2} = 11 \\
\text{equation } h = 11
\]

b) How far is the person above the ground as the wheel starts rotating?

11 metres

c) If it takes 16 seconds for the person to return to the same height as in b), determine the period.

16 seconds = half the period

\[
\text{period } = 32 \text{ seconds}
\]

d) Write the sinusoidal function represented by the graph in the form $h(t) = a \sin bt + d$.

Express the parameters as exact values.

\[
a = 9 \quad b = \frac{2\pi}{32} = \frac{\pi}{16} \quad h(t) = 9 \sin\left(\frac{\pi}{16} t\right) + 11
\]

e) Determine the distance between the person and the ground, to the nearest tenth of a metre, after 30 seconds.

$t = 30 \quad h(t) = 9 \sin\left(\frac{\pi}{16} \times 30\right) + 11 = 7.55... = 7.6 \text{ metres}$

f) How long from the start of the ride does it take for the person to be at a height of 5 metres above the ground? Answer to the nearest tenth of a second.

\[
\text{graph } y_1 = 9 \sin\left(\frac{\pi}{16} x\right) + 11 \quad \text{intersect} \quad y_2 = 5 \quad \text{at } x = 19.716... = 19.7 \text{ seconds}
\]
5. On one particular day in Kylebane harbour, the equation of the sinusoidal function that represents the relationship between the depth of water, \( y \) metres, and the time, \( t \) hours after midnight, is

\[
y = 4 \sin \left( \frac{\pi}{6} t + \frac{\pi}{3} \right) + 15.
\]

a) Calculate the depth of water 4 hours after midnight.

\[
t = 4, \quad y = 4 \sin \left( \frac{\pi \times 4}{6} + \frac{\pi}{3} \right) + 15 = 15 \text{ metres}
\]

b) Determine the maximum and minimum depths of water in the harbour.

\[
\text{amplitude} = 4 \text{ m} \quad \text{midvalue} + \text{amplitude} = 15 + 4 = 19 \\
\text{midvalue} = 15 \text{ m} \quad \text{midvalue} - \text{amplitude} = 15 - 4 = 11
\]

maximum and minimum depths = 19 metres and 11 metres

c) Calculate, to the nearest tenth of a metre, the depth of water at midnight.

\[
t = 0, \quad y = 4 \sin \left( \frac{\pi}{3} \right) + 15 = 18.464... = 18.5 \text{ metres}
\]

d) A large supertanker is in the harbour at midnight. It can remain in the harbour if the depth of water in the harbour is at least 12 metres. Since the minimum depth of water in the harbour is less than 12 metres, the tanker will need to leave the harbour before the depth falls below 12 metres.

**Explain** clearly how to use a graphical approach to determine the latest time, to the whole hour, that the supertanker can remain in the harbour.

State an appropriate graphing calculator window.

Graph the equations \( y_1 = 4 \sin \left( \frac{\pi}{6} x + \frac{\pi}{3} \right) + 15 \) and \( y_2 = 12 \)

in a window like \( x: [0, 24, 4] \) \( y: [0, 20, 5] \).

Use the intersect feature to determine the \( x \)-coordinate of the first point of intersection and round down.

e) Calculate the latest time, to the whole hour, that the supertanker can remain in the harbour.

\[
\text{Intersect at } x = 5.619... \\
\text{round down to } x = 5 \quad 5:00 \text{ a.m.}
\]
The next two parts to the question require a knowledge of permutations and combinations.

f) In Kylebane harbour, there are six berths for cruise ships. The harbourmaster is responsible for allocating ships to the berths.

At midnight, there are no cruise ships in the harbour. Four cruise ships are scheduled to arrive in the morning. In how many different ways can the harbourmaster allocate berths for these ships?

\[6P_4 = 360\]

g) The following morning, the four cruise ships are scheduled to leave the harbour around 10 am. If only one ship can leave the harbour at a time, in how many different ways can the harbourmaster arrange the departure of the cruise ships if the largest ship must leave first?

\[3! = 6\]

6. The alarm in a noisy factory is a siren whose loudness, \(L\) decibels, fluctuates so that \(t\) seconds after starting, the loudness is given by the sinusoidal function

\[L(t) = 18 \sin \left(\frac{\pi}{15} t\right) + 60.\]

a) How can the maximum and minimum loudness of the siren be determined from the function \(L(t)\)? Determine the maximum and minimum loudness.

Amplitude = 18
Midline = 60
Add the amplitude to the midline value to get the maximum value of 60 + 18 = 78 dB.

Subtract the amplitude from the midline value to get the minimum value of 60 - 18 = 42 dB.

b) Explain why the period of the function is 30 seconds.

\[b = \frac{\pi}{15}\]

\[\text{Period} = \frac{2\pi}{b} = \frac{2\pi}{\pi/15} = 30 \text{ seconds}\]

c) Write a suitable window that can be used to display two cycles of the graph of the function.

\[x: [0, 40, 5], \quad y: [30, 100, 10]\]

d) After how many seconds, to the nearest tenth, does the loudness first reach 70 decibels?

\[y_2 = 70, \quad x = 2.812\ldots\quad 2.8 \text{ seconds}\]

e) The background noise level in the factory is 45 decibels. Between which times in the first cycle, to the nearest tenth of a second, is the alarm siren at a lower level than the background noise?

\[y_3 = 45, \quad \text{intersects at} x = 19.703\ldots \text{ and} 25.296\ldots\]

f) For what percentage of each 30 second cycle, to the nearest per cent, is the alarm siren audible over the background factory noise?

\[25.3 - 19.7 = 5.6 \text{ seconds}\]

\[\frac{5.6}{30} = 0.186\ldots = 18.6\ldots\%\]

\[100\% - 18.6\% = 81.3\ldots\%\]

\[81\%\]
Use the following information to answer the next two questions.

The graph below shows how the number of hours \((h)\) of daylight in a town changes during the year.

\[ h = \frac{7}{3} \sin\left(\frac{2\pi}{365} x\right) + \frac{35}{3} \]

April 21

365

April 21

7. Mid-winter is the day with the least hours of daylight. The number of hours of daylight, to the nearest hundredth of an hour, that there will be on mid-winter’s day is \(91.3\) hours.

(Record your answer in the numerical response box from left to right.)

Minimum point on the graph is at \(x = 273.75\) and \(y = 9.33\ldots\)

Minimum value: \(9.33\ldots = 9.3\) hours

8. The number of days after April 21 that mid-winter occurs is \(274\).

(Record your answer in the numerical response box from left to right.)

\[ x = 273.75 \approx 274 \]