8.2 Exploring Graphs of Periodic Functions

Keep in Mind

- The functions $y = \sin x$ and $y = \cos x$ are periodic.
- The graphs of these two periodic functions have the following common characteristics:
  - multiple $x$-intercepts
  - one $y$-intercept
  - a domain of $\{x \mid x \in \mathbb{R}\}$
  - a range of $\{y \mid -1 \leq y \leq 1, y \in \mathbb{R}\}$
  - an amplitude of 1
  - a period of $360^\circ$, or $2\pi$
  - a midline defined by the equation $y = 0$. This midline is the horizontal line halfway between the maximum and minimum values. The two graphs oscillate about this line.
- The graphs of $y = \sin x$ and $y = \cos x$ are congruent curves. That is, if all the points on a cosine curve are translated to the right by $90^\circ$, or $\frac{\pi}{2}$, the result is a sine curve.
- The cycle of a graph is one complete unit in the repeating pattern of the graph.
- The period of a graph is the length of one cycle.

Example

Emily says that this graph is a graph of $y = \sin x$. Is she correct? Explain.
Solution

Step 1. I compared the graph's characteristics to those of $y = \sin x$.

<table>
<thead>
<tr>
<th>The Graph of $y = \sin x$</th>
<th>Emily's Graph</th>
</tr>
</thead>
<tbody>
<tr>
<td>• has multiple $x$-intercepts</td>
<td>Yes; the graph crosses the $x$-axis more than once.</td>
</tr>
<tr>
<td>• has one $y$-intercept</td>
<td>Yes; it crosses the $y$-axis just once.</td>
</tr>
<tr>
<td>• has a domain of ${x \mid x \in \mathbb{R}}$</td>
<td>Yes; the graph can continue along the $x$-axis in either direction.</td>
</tr>
<tr>
<td>• has a range of $(-1 \leq y \leq 1, y \in \mathbb{R})$</td>
<td>Yes; the graph oscillates between $-1$ and $1$.</td>
</tr>
<tr>
<td>• has an amplitude of $1$</td>
<td>Yes; the amplitude is $1$.</td>
</tr>
<tr>
<td>• has a period of $360^\circ$ or $2\pi$</td>
<td>No; the period of this graph is $180^\circ$.</td>
</tr>
</tbody>
</table>

Step 2. I considered the results.

Emily's graph is not a graph of $y = \sin x$ because the periods are not the same.

Practice

1. Is each graph a graph of $y = \sin x$, $y = \cos x$, or neither? Briefly explain your answer.

   a) $y = \cos x$

   b) $y = \cos x$; neither, amp is greater than $1$

   c) neither — a parabola

   d) neither; period is $720^\circ$