8.1 Understanding Angles

Today's Focus: Estimate and determine benchmarks for angle measure

**DEFINITION**

Define SINUSOIDAL FUNCTION and give an example.

Any **PERIODIC** function whose graph has the same shape as \( y = \sin(x) \)

\[ y = a \sin(b(x - c)) + d \]

\[ y = a \cos(b(x - c)) + d \]

*Note: Periodic means a graph that repeats.*

**INVESTIGATION**

1. Draw a circle with a diameter of 10cm. Label the radius as \( CA \).

![Circle with radius labeled CA and diameter 10cm](image)

2. Cut a pipe cleaner to a length equal to radius \( CA \) for each circle.
3. Place and bend the pipe cleaner (the one from step 2 above) around the circumference of your circle, beginning at point A and ending at point P, as shown by the arrow arc below:

4. On the circle you are working with, draw radius CP.

5. Measure \( \angle PCA \) with a protractor. What is the measure of \( \angle PCA \) in degrees?

\[ \sim 58^\circ = \text{1 radian} \]

6. Use the pipe cleaner to continue marking radius lengths all the way around your circle. About how many radius lengths are there in one complete circumference of your circle?

\[ 360^\circ = 6.2 \text{ rad.} \]

7. The circumference, \( C \), of a circle is given by the formula \( C = 2\pi r \), where \( r \) represents the radius. Explain how this formula relates to your answer for part 6.

\[ C = 2\pi r \quad \frac{C}{r} = 2\pi = 6.28 \text{ rad or } 360^\circ \quad \therefore \quad 2\pi = 360^\circ \]

8. What does the circumference formula find for us? What degree measure is equal to \( 2\pi \) radians?

\[ 2\pi = 360^\circ \]

KEY IDEAS!!!

- Radian measure is an alternative way to express the size of an angle.
- Using radians allows you to express the measure of an angle as a real number without units.
- The central angle formed by one complete revolution in a circle is \( 360^\circ \), or \( 2\pi \) in radian measure.
THINGS TO REMEMBER!!!

- Use benchmarks to estimate the degree measure of an angle given in radians.
- In radian measure,
  - 1 is equivalent to about 60°;
  - π is equivalent to 180°;
  - 2π is equivalent to 360°.
- Decimal approximations can be used for benchmarks to visualize the approximate size of an angle measured in radians.

\[ \angle \theta = 57.3^\circ \text{ or } 1 \text{ in radian measure} \]

**Example 1:** Estimate the value of each angle in radian measure:

a) \[ 90^\circ = \frac{360^\circ}{4} = \frac{\pi}{2} \text{ rad} \]

b) \[ 120^\circ = \frac{1}{3} \text{ of } 360^\circ = \frac{2\pi}{3} \text{ rad} \]

**Example 2:** Estimate the value of each angle in radian measure.

a) \[ 240^\circ = \frac{2}{3} \text{ of } 360^\circ = \frac{2\pi}{3} \text{ rad} \]

b) \[ 450^\circ = 360^\circ + 90^\circ = \frac{5}{4} \text{ of } 360^\circ = \frac{5\pi}{4} \approx 4.88 \text{ rad} \]
Radian measure is an alternate way to express the size of an angle. Using radians allows you to express the measure of an angle as a real number without units.

**Conversion Chart**

\[
\frac{\pi}{180} \quad \frac{\pi}{360} \quad \frac{180}{\pi}
\]

Degrees to Radians multiply by \(\frac{\pi}{180}\) \quad Radians to Degrees multiply by \(\frac{180}{\pi}\)

**Example 3:** Change each degree to radian and each radian to degree:

a. \(30^\circ\)

\[
30^\circ \times \frac{\pi}{180} = \frac{\pi}{6} = 0.52 \text{ rad}
\]

b. \(\frac{2\pi}{3}\)

\[
\frac{2\pi}{3} \times \frac{180}{\pi} = 120^\circ
\]

c. \(210^\circ\)

\[
210^\circ \times \frac{\pi}{180} = \frac{7\pi}{6} = 3.7 \text{ rad}
\]

d. \(\frac{9\pi}{5}\)

\[
\frac{9\pi}{5} \times \frac{180}{\pi} = 324^\circ
\]