Applications of Exponents and Logarithms Lesson #9: Logarithmic Scales and Applications

Earthquakes

The Richter scale, named after the American seismologist Charles Richter (1900-85), is used to measure the magnitude of an earthquake. The magnitude of an earthquake is a measure of the amount of energy released. It is determined from the logarithm of the amplitude of waves recorded by seismographs.

The Richter scale is logarithmic – a difference in one unit in magnitude corresponds to a factor of ten difference in intensity. This means that each whole number step represents a ten-fold increase in intensity. Therefore, a magnitude 9 earthquake is ten times larger than a magnitude 8 earthquake, one hundred times larger then a magnitude 7 earthquake, and one thousand times larger than a magnitude 6 earthquake.

A magnitude of 1.0 on the Richter Scale is equivalent to a large blast at a construction site using approximately 14 kg of TNT. It is 10 times as intense as the zero reference point. An earthquake with magnitude 2.0 is $10^2$ times as intense as the zero reference point, etc.

Any earthquakes having magnitudes in excess of 6.0 are considered dangerous. The largest yet recorded, the Chilean earthquake of 1960, registered 9.5 on the Richter scale. The most powerful one recorded in North America, the Alaska quake of 1964, reached 8.4 on the Richter scale.

The Richter Scale

<table>
<thead>
<tr>
<th>Magnitude</th>
<th>Event</th>
</tr>
</thead>
<tbody>
<tr>
<td>9.5</td>
<td>Chile 1960</td>
</tr>
<tr>
<td>9.2</td>
<td>Alaska 1964</td>
</tr>
<tr>
<td>9.0</td>
<td>Japan 2011</td>
</tr>
<tr>
<td>8.9</td>
<td>Japan 1933</td>
</tr>
<tr>
<td>8.1</td>
<td>Mexico 1985</td>
</tr>
<tr>
<td>7.9</td>
<td>San Francisco 1906</td>
</tr>
<tr>
<td>7.8</td>
<td>Chile 2005</td>
</tr>
<tr>
<td>7.5</td>
<td>Guatemala 1976</td>
</tr>
<tr>
<td>7.3</td>
<td>Vancouver Island 1946</td>
</tr>
<tr>
<td>7.0</td>
<td>Turkey 1966</td>
</tr>
<tr>
<td>6.9</td>
<td>Italy 1976</td>
</tr>
<tr>
<td>6.3</td>
<td>Peru 2012</td>
</tr>
<tr>
<td>6.0</td>
<td>Columbia 1983</td>
</tr>
</tbody>
</table>

Complete the following:

a) An earthquake of magnitude 8 is $10^3$ times as intense as an earthquake of magnitude 7.

b) An earthquake of magnitude 7 is $10,000$ times as intense as an earthquake of magnitude 3.

c) An earthquake of magnitude 4 is $\frac{1}{100}$ times as intense as an earthquake of magnitude 6.

d) An earthquake of magnitude 4.8 is $\frac{1}{100}$ times as intense as an earthquake of magnitude 6.8.

e) The 1960 earthquake in Chile was $1000$ times as intense as the 1976 earthquake in Italy and $10,000$ times as intense as the 1983 earthquake in Colombia.

f) The 1966 earthquake in Turkey was $\frac{1}{100}$ as intense as the 1933 earthquake in Japan.
Comparing Earthquake Intensities

We have seen how to compare the intensities of earthquakes whose magnitudes differ by an integer. How would we compare the intensities of the 1960 earthquake in Chile (magnitude 9.5) and the 2005 earthquake in Chile (magnitude 7.8)?

To compare the intensities of magnitude \( M_1 \) and \( M_2 \), consider the following:

The magnitude of an earthquake is given by the formula \( M = \log \left( \frac{I}{I_0} \right) \), where \( I \) is the earthquake intensity and \( I_0 \) is a reference intensity, e.g., \( M_1 = \log \left( \frac{I_1}{I_0} \right) \) and \( M_2 = \log \left( \frac{I_2}{I_0} \right) \).

a) We can use the laws of logarithms to show that \( M_1 - M_2 = \log \left( \frac{I_1}{I_2} \right) \).

Complete the work shown below.

\[
M_1 - M_2 = \log \left( \frac{I_1}{I_0} \right) - \log \left( \frac{I_2}{I_0} \right) = [\log I_1 - \log I_0] - [\log I_2 - \log I_0]
\]

\[
= \log I_1 - \log I_2 = \log \left( \frac{I_1}{I_2} \right).
\]

Converting the equation \( M_1 - M_2 = \log \left( \frac{I_1}{I_2} \right) \) to exponential form yields the result \( \frac{I_1}{I_2} = 10^{M_1 - M_2} \), a formula which can be used to compare the intensities of two earthquakes, given their magnitudes.

The intensities of two earthquakes can be compared using \( \frac{I_1}{I_2} = 10^{M_1 - M_2} \).

How many times more intense was the 1960 earthquake in Chile (magnitude 9.5) than the 2005 earthquake in Chile (magnitude 7.8)? Answer to the nearest whole number.

\[
\frac{I_1}{I_2} = 10^{9.5-7.8} = 10^{1.7} \approx 50.11...
\]

50 times more intense
A major earthquake of magnitude 7.5 is 375 times as intense as a minor earthquake.

a) Is the magnitude of the minor earthquake greater or smaller than 7.5?

b) Use the formula \( \frac{I_1}{I_2} = 10^{M_1 - M_2} \) to determine, to one decimal place, the magnitude of the minor earthquake.

\[
\begin{align*}
375 & = 10^{7.5 - x} \\
\log_{10} 375 & = 7.5 - x \\
x & = 7.5 - \log_{10} 375 \approx 4.925...
\end{align*}
\]

Magnitude 4.9

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**Loudness of Sound**

The loudness of a sound was originally measured in **Bels**, named after Alexander Graham Bell.

The current unit used is the **decibel (dB)**, which is equal to one tenth of a Bel.

The Bel scale, like the Richter scale, is logarithmic – a difference of 1 Bel, or 10 decibels, corresponds to a factor of ten difference in sound intensity.

A leaf rustling (10 decibels or 1 Bel) is 10 times as intense as the threshold of hearing.

A whisper (30 decibels or 3 Bels) is \(10^3\), or 1000 times as intense as the threshold of hearing and \(10^2\), or 100 times as intense as a leaf rustling.

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**The Decibel (dB) Scale**

<table>
<thead>
<tr>
<th>dB</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>150</td>
<td>Jet Engine at 20 m</td>
</tr>
<tr>
<td>125</td>
<td>Threshold of Pain</td>
</tr>
<tr>
<td>100</td>
<td>Rock Concert</td>
</tr>
<tr>
<td>75</td>
<td>Power Saw</td>
</tr>
<tr>
<td>50</td>
<td>Train Whistle at 150 m</td>
</tr>
<tr>
<td>25</td>
<td>Telephone Dial Tone</td>
</tr>
<tr>
<td>0</td>
<td>Piano Playing Conversation</td>
</tr>
<tr>
<td>0</td>
<td>Whisper</td>
</tr>
<tr>
<td>0</td>
<td>Leaf Rustling</td>
</tr>
<tr>
<td>0</td>
<td>Threshold of Hearing</td>
</tr>
</tbody>
</table>

---

Complete the following using the chart above.

a) A power saw (120 dB) is \( 10000 \) times as intense as a telephone dial tone (80 dB).

\[
\begin{align*}
120 \text{ dB} & = 12 \text{ Bels} \\
80 \text{ dB} & = 8 \text{ Bels}
\end{align*}
\]

b) A jet engine at 20 m (145 dB) is \( 100 \) times as intense as the threshold of pain (125 dB).

\[
\begin{align*}
145 \text{ dB} & = 14.5 \text{ Bels} \\
125 \text{ dB} & = 12.5 \text{ Bels}
\end{align*}
\]

c) A whisper (30 dB) is \( \frac{1}{1000} \) times as intense as a conversation (60 dB).

\[
\begin{align*}
30 \text{ dB} & = 3 \text{ Bels} \\
60 \text{ dB} & = 6 \text{ Bels}
\end{align*}
\]

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Comparing Intensities of Sounds

We have seen how to compare the intensities of sounds whose Bel levels differ by an integer. To compare the intensities of sounds when the difference in the Bel levels is not an integer, use the following:

For Bels

\[ \text{Bels} = \log\left( \frac{I}{I_0} \right) \]

To compare the intensities of two sounds measured in Bels, we use

\[ \frac{I_1}{I_2} = 10^{B_1 - B_2} \]

For dB

\[ \text{dB} = 10 \log\left( \frac{I}{I_0} \right) \]

To compare the intensities of two sounds measured in decibels, we use

\[ \frac{I_1}{I_2} = 10^{\frac{\text{dB}_1 - \text{dB}_2}{10}} \]

We can derive the above formulas in a way similar to the one used in the previous section.

Class Ex. #5

How many times more intense is the sound of a piano playing (67 dB) than a whisper (22 dB)?

\[ \frac{I_1}{I_2} = 10^{\frac{67 - 22}{10}} = 10^{4.5} = 31623 \text{ times more intense} \]

Class Ex. #6

Given that dB = 10 \log\left( \frac{I}{I_0} \right), show that the decibel level for the threshold of pain (125 dB) has a sound intensity \(10^{12.5}I_0\).

\[ 12.5 = 10 \log\left( \frac{I}{I_0} \right) \]

\[ 10^{12.5} = \frac{I}{I_0} \]

\[ \mathcal{I} = 10^{12.5} \mathcal{I}_0 \]

Complete Assignment Questions #1 - #7
**pH Scale**

In 1909, Sören Sörenson, a Dutch chemist, introduced the term pH, representing the expression "the power of hydrogen", to measure the extremely wide range of hydrogen ion concentration in substances.

The pH scale measures the range of hydrogen ion concentration by determining the acidity or the alkalinity of a solution. pH values below 7 represent increasing acidity and pH values above 7 represent increasing alkalinity. The value 7 represents the neutral level on the pH scale, where the solution is neither acidic nor alkaline.

![PH Scale Chart]

Similar to the Richter scale, the pH scale is logarithmic – a difference in one unit of pH corresponds to a factor of ten difference in intensity. This means that each whole number step represents a ten-fold change in intensity.

Therefore, if vinegar has a pH of 3, then it is ten times as acidic as tomato juice (pH of 4) and one hundred times as acidic as normal rain (pH of 5).

On the other hand, household ammonia (pH of 11.5) is ten times as alkaline as milk of magnesia (pH of 10.5) and one thousand times as alkaline as sea water (pH of 8.5).

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**Complete the following, using the approximate pH values given.**

a) Tomato juice is \(10^4\) times as acidic as pure water.

b) Eggs are \(10^3\) times as alkaline as pure water.

c) Eggs are \(\frac{1}{10}\) times as acidic as pure water.

d) Milk of magnesia is 1000 times as alkaline as blood.

e) Vinegar is 100 times as acidic as normal rain.

f) Eggs are \(\frac{1}{10000}\) times as alkaline as washing soda.

---

<table>
<thead>
<tr>
<th>Solution</th>
<th>pH</th>
</tr>
</thead>
<tbody>
<tr>
<td>Battery Acid</td>
<td>0.5</td>
</tr>
<tr>
<td>Lemon Juice</td>
<td>2.5</td>
</tr>
<tr>
<td>Vinegar</td>
<td>3</td>
</tr>
<tr>
<td>Tomato Juice</td>
<td>4</td>
</tr>
<tr>
<td>Normal Rain</td>
<td>5</td>
</tr>
<tr>
<td>Pure Water</td>
<td>7</td>
</tr>
<tr>
<td>Blood</td>
<td>7.5</td>
</tr>
<tr>
<td>Eggs</td>
<td>8</td>
</tr>
<tr>
<td>Milk of Magnesia</td>
<td>10.5</td>
</tr>
<tr>
<td>Washing Soda</td>
<td>12</td>
</tr>
</tbody>
</table>
Comparing Acidity and Alkalinity of Solutions

To find how much more acidic or alkaline one solution is to another, use $10^{pH_1 - pH_2}$.

Pure water, swimming pool water, and sea water have pH levels of 7, 7.5, and 8.4 respectively.

a) Sea water is how many times as alkaline as pure water?

$$10^{8.4-7} = 10^{1.4} = \text{25 times}$$

b) Sea water is how many times as alkaline as swimming pool water?

$$10^{8.4-7.5} = 10^{0.9} = \text{8 times}$$

Formula for pH

The pH of a solution is defined as $pH = -\log [H^+]$, where $[H^+]$ is the hydrogen ion concentration (expressed in moles/litre).

A patient gave a urine sample which was found to have a pH of 5.7. What was the hydrogen ion concentration? Answer in scientific notation using one decimal place.

$$5.7 = -\log [H^+]$$

$$-5.7 = \log [H^+]$$

$$10^{-5.7} = [H^+] = 2.0 \times 10^{-6} \text{ moles/litre}$$

Determine the pH of a solution, to the nearest tenth, if the hydrogen ion concentration is $3.4 \times 10^{-4}$ mol/L.

$$pH = -\log (3.4 \times 10^{-4})$$

$$pH = 3.5$$

Complete Assignment Questions #8 - #13

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Assignment

1. How many times more intense was the 2011 earthquake in Japan (magnitude 9.0) than the 2012 earthquake in Peru (magnitude 6.3)? Answer to the nearest whole number.

\[ \frac{10^{9.0-6.3}}{10^{2.7}} = 501.18 \ldots \quad \text{501 times} \]

2. At 2:45 pm on March 11, 2011, a major earthquake of magnitude 9.0 hit the east coast of Japan. Half an hour later, a second earthquake of magnitude 7.9 hit the same region. How many times more intense, to the nearest whole number, was the first earthquake than the second one?

\[ \frac{10^{9.0-7.9}}{10^{1.1}} = 12.58 \ldots \quad \text{13 times} \]

3. An earthquake of magnitude 6.4 is twice as intense as a second earthquake. Determine the magnitude of the second earthquake.

\[ \frac{I_1}{I_2} = 10^{M_1-M_2} \quad \quad x = 6.4 - \log 2 \]

\[ x = 6.098 \ldots \quad \text{Magnitude 6.1} \]

4. An earthquake in Peru had a magnitude of 7.7 on the Richter Scale. The following day, a second earthquake with one-third of the intensity of the first hit the same region. Determine the magnitude of the second earthquake to the nearest tenth.

\[ \frac{I_2}{I_1} = 10^{M_2-M_1} \quad \quad x = 7.7 + \log 10^{\frac{1}{3}} \]

\[ x = 7.22 \ldots \quad \text{Magnitude 7.2} \]

5. How many times more intense is the sound of a referee’s whistle (125 dB) than a train whistle at 200 m (90 dB)? Answer to the nearest whole number.

\[ \frac{I_1}{I_2} = 10^{\frac{125-90}{10}} = 10^{3.5} = 3162 \quad \text{times} \]

6. Compare the sound intensities of a clarinet (95 dB) and a flute (89 dB).

\[ \frac{I_1}{I_2} = 10^{\frac{95-89}{10}} = 10^{0.6} = 4 \quad \text{The sound of the clarinet is 4 times as intense as the sound of the flute.} \]

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The sound level of a telephone ringing is 80 dB. When the fire alarm goes off, the ring of the telephone is not heard because the sound intensity of the fire alarm is 12 times as intense as the sound intensity of the telephone.

In order to determine the sound level of the fire alarm, Ahmed did the following calculation:

Line 1: \[ 12 = 10^{\frac{80-x}{10}} \]

Line 2: \[ \log_{10} 12 = \log_{10} 10^{\frac{80-x}{10}} \]

Line 3: \[ \log_{10} 12 = \frac{80-x}{10} \]

Line 4: \[ 1.08 = \frac{80-x}{10} \]

Line 5: \[ 10.8 = 80 - x \]

Line 6: \[ x = 80 - 10.8 \]

Line 7: \[ x = 69.2 \]

7. a) Without looking at his work, how can we tell that Ahmed’s answer of 69.2 dB cannot be correct?

The fire alarm is louder, so the sound level of the fire alarm should be more than the sound level of the telephone ring (80 dB), not less.

b) Explain where Ahmed made his error and determine the sound level of the fire alarm.

Line 1 should be \[ 12 = 10^\frac{x-80}{10} \] or \[ \frac{1}{12} = 10^\frac{80-x}{10} \].

\[ \log_{10} 12 = \frac{x-80}{10} \]

\[ 10 \log_{10} 12 = x - 80 \]

\[ 10 \log_{10} 12 + 80 = x \]

Sound level: 90.8 dB

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8. Use the chart to answer the following to the nearest whole number.
   a) Eggs are how many times as alkaline as blood?
      \[ 10^{8.7-7.5} = 10^{1.2} = 3.162 \]
      \[ \text{3.2 times} \]
   b) Black coffee is how many times as acidic as milk?
      \[ 10^{6.6-5.1} = 10^{1.5} = 31.62 \]
      \[ \text{3.2 times} \]

9. A river has a pH value of 6.4 upstream from a chemical factory and a pH value of 5.8 downstream of the factory. Compare the acidity levels.
   \[ 10^{6.4-5.8} = 10^{0.6} = 3.98... \]
   \[ \text{Downstream is 4 times as acidic as upstream.} \]

10. The pH of a solution is defined as \( pH = -\log [H^+] \), where \([H^+]\) is the hydrogen ion concentration (expressed as \text{mol/L}).
   a) If a solution has a hydrogen ion concentration of \(1.21 \times 10^{-2}\) \text{mol/L},
      determine the pH of the solution to the nearest tenth.
      \[ pH = -\log(1.21 \times 10^{-2}) = 1.917 \]
      \[ pH = 1.9 \]
   b) A vinegar solution has a pH of 3.2. Determine its hydrogen ion concentration in scientific notation to one decimal place.
      \[ 3.2 = -\log[H^+] \]
      \[ -3.2 = \log[H^+] \]
      \[ [H^+] = 10^{-3.2} = 6.309 \times 10^{-4} \]
      \[ \text{concentration is} \quad 6.3 \times 10^{-4} \text{ mol/L} \]
   c) A weaker vinegar solution is 25% as acidic as the solution in b).
      Determine its pH value to the nearest tenth.
      \[ \frac{25}{100} = 10^{3.2-x} \]
      \[ x = 3.2 - \log_{10} 10^{-4} \]
      \[ x = 3.802... \]
      \[ \text{pH} = 3.8 \]

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The equation that defines the decibel level for any sound is
\[ \text{dB} = 10 \log \left( \frac{I}{I_0} \right) \]
where \( I = \) intensity of sound being measured
\( I_0 = \) intensity of sound at the threshold of hearing

11. The sound of a page being turned is 10,000 times as intense as \( I_0 \).
The loudness of a page being turned is
A. 4 decibels  \( dB = 10 \log (10000) \)
B. 5 decibels  \( = 100 \times 4 \)
C. 14 decibels  \( = 40 \)
D. 40 decibels

12. Solution A has a pH of 5.4. Solution B is 100 times as acidic as Solution A.
To the nearest tenth, the pH of Solution B is \( \boxed{7.4} \).

13. A major earthquake of magnitude 8.2 is 110 times as intense as a minor earthquake.
The magnitude, to the nearest tenth, of the minor earthquake is \( \boxed{6.2} \).

**Answer Key**

1. 501  2. 13  3. 6.1  4. 7.2  5. 3162
6. The sound of the clarinet is 4 times as intense as the sound of the flute.
7. a) The fire alarm is louder, so the sound level of the fire alarm should be more than the sound level of the telephone ring (80 dB), not less.
   b) Line 1 should be \( \frac{17}{12} = 10^{\frac{80-80}{10}} \) or \( 10^{\frac{1}{12}} = 10^{\frac{80-x}{10}} \), 90.8 dB
8. a) 3  b) 32
9. Downstream is 4 times as acidic as upstream.
10.a) 1.9  b) \( 6.3 \times 10^{-4} \) mol/L  c) 3.8
11. 12. \( \boxed{3.4} \)
13. 6.2