6.5 Financial Applications

Today's Focus: Solve problems that involve loans and mortgages by using exponential functions

KEY IDEAS!!!

- Financial institutions charge compound interest on all loans and pay compound interest on some investments. Compound interest is calculated at regular intervals by first adding any accumulated interest to the principal and then multiplying the result by the interest rate.
- When money earns or is charged compound interest, an exponential function can be used to model the situation.

THINGS TO REMEMBER!!!

Compound interest can be calculated using the formula below, the TVM solver on the graphing calculator (TI-83+), or using a spreadsheet.

\[ A = P \left(1 + \frac{r}{n}\right)^{nt}\]

**Formula:** \(A = P \left(1 + \frac{r}{n}\right)^{nt}\)  
A = Accumulated Amount

**NOTE:** This formula is NOT on your formula Sheet for the Diploma!

- **P** = principal
- **r** = annual interest rate
- **n** = number of compounding periods per year
- **t** = number of years

Common Compounding Periods:

- **Annual** 1/year
- **Semi annual** 2/year (every 6 months)
- **Quarterly** 4/year (every 3 months)
- **Monthly** 12/year (every month)
- **Daily** 365/year (every day)
Example 1: Andrew invests $100. Determine the values of P, A, r, n and t for each situation.
- An annual interest rate of 4%, compounded semi-annually, for 3 years.

\[
P = 100 \\
r = 0.04 \\
\text{n} = 2 \\
t = 3 \\
A = ?
\]

\[
A = P \left(1 + \frac{r}{n}\right)^{nt} \\
A = 100 \left(1 + \frac{0.04}{2}\right)^{2 \times 3} \\
A = 100 \left(1.02\right)^{6} = 112.61
\]

- An annual interest rate of \(6\frac{3}{8}\)%, compounded monthly for 2 years.

\[
r = 0.06375 \\
\text{n} = 12 \\
t = 2 \\
P = 100 \\
A = ?
\]

\[
A = 100 \left(1 + \frac{0.06375}{12}\right)^{2 \times 12} \\
A = 100 \left(1.0053125\right)^{24} \\
A = 113.56
\]

Example 2: Adam invests $500 at 8% per annum, compounded annually, for 2 years. Determine the accumulated amount at the end of the year.

\[
P = 500 \\
r = 0.08 \\
\text{n} = 1 \\
t = 2 \\
A = ?
\]

\[
A = P \left(1 + \frac{r}{n}\right)^{nt} \\
A = 500 \left(1 + \frac{0.08}{1}\right)^{2} \\
A = 500 \left(1.08\right)^{2} \\
A = 583.20
\]

Example 3: Suppose Will invests $1000 for 10 years. If he earns an annual interest rate of 7.75%, compounded monthly, determine the total interest he earned over the 10 years.

\[
P = 1000 \\
t = 10 \\
r = 0.0775 \\
\text{n} = 12 \\
A = ?
\]

\[
A = P \left(1 + \frac{r}{n}\right)^{nt} \\
A = 1000 \left(1 + \frac{0.0775}{12}\right)^{10 \times 12} \\
A = 2165.19
\]

Interest = 2165.19 - 1000 = $1165.19
Example 4: Michael has an opportunity to invest some money for 5 years compounded quarterly at 6% annual interest. How much does he need to invest so that he has $5000 in five years?

\[ A = P \left(1 + \frac{r}{n}\right)^{nt} \]

\[ 5000 = P \left(1 + \frac{0.06}{4}\right)^{5 \times 4} \]

\[ 5000 = P \left(1.06\right)^{20} \]

\[ \frac{5000}{1.3382} = \frac{P(1.3382)}{1.3382} \]

\[ P = \frac{3736.29}{1.3382} \]

Michael would have to invest $3736.29

Example 5: Megan needs $25 000 in eight years for a down payment on a house. The bank is offering 5.625% interest per annum compounded monthly for 8 years. How much does Megan need to put away today to have $25 000 in eight years?

\[ A = P \left(1 + \frac{r}{n}\right)^{nt} \]

\[ 25000 = P \left(1 + \frac{0.05625}{12}\right)^{8 \times 12} \]

\[ 25000 = P \left(1.004666\right)^{96} \]

\[ \frac{25000}{1.56666} = \frac{P(1.56666)}{1.56666} \]

\[ P = \frac{15957.47}{1.56666} \]

She would have to invest $15957.47