6.5 Financial Applications Involving Exponential Functions

Keep in Mind

- Compound interest is charged on most loans and paid on some investments.
  To calculate compound interest, add any accumulated interest to the amount invested or borrowed (the principal) and multiply the result by the interest rate.

- To model compound interest, use $A(n) = P(1 + i)^n$, where
  - $A$ represents the value of the investment/loan at a given time (the future value)
  - $P$ represents the principal
  - $i$ represents the interest rate per compounding period, as a decimal
  - $n$ represents the number of compounding periods

- To determine the interest rate per compounding period, divide the annual rate by the number of times interest is paid.

<table>
<thead>
<tr>
<th>Compounding Period</th>
<th>Number of Times Interest Is Paid</th>
<th>Interest Rate per Compounding Period, $i$</th>
<th>For Example: 4.8%/Year</th>
</tr>
</thead>
<tbody>
<tr>
<td>daily</td>
<td>365 times per year</td>
<td>$i = \frac{\text{annual rate}}{365}$</td>
<td>0.048 $\frac{365}{365} = 0.00013...$</td>
</tr>
<tr>
<td>weekly</td>
<td>52 times per year</td>
<td>$i = \frac{\text{annual rate}}{52}$</td>
<td>0.048 $\frac{52}{52} = 0.00092...$</td>
</tr>
<tr>
<td>bi-weekly</td>
<td>24 times per year</td>
<td>$i = \frac{\text{annual rate}}{24}$</td>
<td>0.048 $\frac{24}{24} = 0.002$</td>
</tr>
<tr>
<td>monthly</td>
<td>12 times per year</td>
<td>$i = \frac{\text{annual rate}}{12}$</td>
<td>0.048 $\frac{12}{12} = 0.004$</td>
</tr>
<tr>
<td>quarterly</td>
<td>4 times per year</td>
<td>$i = \frac{\text{annual rate}}{4}$</td>
<td>0.048 $\frac{4}{4} = 0.012$</td>
</tr>
<tr>
<td>semi-annually</td>
<td>2 times per year</td>
<td>$i = \frac{\text{annual rate}}{2}$</td>
<td>0.048 $\frac{2}{2} = 0.024$</td>
</tr>
<tr>
<td>annually</td>
<td>1 time per year</td>
<td>$i = \frac{\text{annual rate}}{1}$</td>
<td>0.048 $\frac{1}{1} = 0.048$</td>
</tr>
</tbody>
</table>

Copyright © 2012 by Nelson Education Ltd.
Example

Sheldon invested $3500 in an account that pays 4.8% interest, compounded monthly. This table gives the value of his investment at the end of the first 5 months.

\[ \begin{array}{|c|c|} 
\hline
\text{Time (months)} & \text{Value of Investment (}$) \\
\hline
0 & 3500.00 \\
1 & 3514.00 \\
2 & 3528.056 \\
3 & 3542.168224 \\
4 & 3556.3368969 \\
5 & 3570.56224448 \\
\hline
\end{array} \]

a) Use exponential regression to determine the compound interest function that models this situation.

b) How long, in months, will it take Sheldon’s investment to grow to $3800?

Solution

Step 1. I plotted the scatter plot of time versus value. Then I used exponential regression to determine the function.

\[ \text{Step 2. I performed the exponential regression.} \]

\[ a) \text{ The compound interest formula is } A = 3500(1.004)^t. \]

\[ \text{Step 3. I determined when the investment would grow to } $3800. \]

The point of intersection for the graphs
\[ f(x) = 3500(1.004)^x \text{ and } f(x) = 3800 \]

is (20.600..., 3800).

b) Sheldon’s investment will grow to $3800 after 20.6 months; that is, in the 21st month.
Practice

1. Francis invested money in two different accounts that pay interest compounded annually. His investments can be modelled by the following growth functions, where $x$ represents the number of years:
   I. \( y = 2000(1.06)^x \)
   II. \( y = 3500(1.025)^x \)
   a) What principal did Francis invest in each account?
      - I. $2000
      - II. $3500
   b) State the annual interest rate for each investment.
      - I. \( 1.06 - 1 = 0.06 = 6\% \)
      - II. \( 1.025 - 1 = 0.025 = 2.5\% \)
   c) Determine the value of each investment at the end of 5 years.
      - I. \( y = 2000(1.06)^5 = 2676.45 \)
      - II. \( y = 3500(1.025)^5 = 3959.93 \)
   d) How long, in years, did it take for each account to contain $4000?
      - I. \( x = \frac{\log(4000/2000)}{\log(1.06)} = 11.89 \text{ yrs} \)
      - II. \( x = \frac{\log(4000/3500)}{\log(1.025)} = 5.41 \text{ yrs} \)

2. Feather invested $4000 in a bank account that pays compound interest annually. Her bank gave her yearly figures for her investment over the next few years, as shown in the table.

<table>
<thead>
<tr>
<th>Time (years)</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value of Investment ($)</td>
<td>4000</td>
<td>4023.33</td>
<td>4046.80</td>
<td>4070.41</td>
<td>4094.15</td>
</tr>
</tbody>
</table>

   a) Use these values to create an exponential regression function that models the investment.
      \[ y = 4000(1.0058)^x \]

   b) What is the future value of the investment after 5 years?
      \[ y = 4000 (1.0058)^5 = 4118.03 \]

3. The price of a particular style of sweater is shown over the course of several years. Use exponential regression to estimate the price of the sweater in Year 5.

<table>
<thead>
<tr>
<th>Time (years)</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost of Sweater ($)</td>
<td>65.00</td>
<td>68.25</td>
<td>70.50</td>
<td>73.75</td>
<td>76.25</td>
<td>80.00</td>
</tr>
</tbody>
</table>

   \[ y = 65.73(1.035)^x \]
   \[ y = 65.73(1.035)^5 = 78.22 \]
4. Barnabas has just graduated from college. The interest-free period of his student loan has come to an end. Barnabas must now pay $400 per month at an interest rate of 4.8%, compounded monthly.

a) The bank manager gave Barnabas this equation to determine how long it would take to pay off his loan.

\[(1.004)^{-n} = 0.75\]

Determine the number of months Barnabas will take to pay off the loan.

\[y_1 = (1.004)^{-n}\]

\[y_2 = 0.75\]

\[n = 72 \text{ months}\]

b) What is the total amount that Barnabas will end up paying the bank? Explain.

\[\$400 \times 72 = \$28,800\]

5. Emily took out a loan to buy an oven. She does not have to make any payments until the 5th year. The table shows how much she will owe at the end of each of the next 4 years.

<table>
<thead>
<tr>
<th>End of Year</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Amount Owed ($)</td>
<td>3075.00</td>
<td>3151.88</td>
<td>3230.67</td>
<td>3311.44</td>
</tr>
</tbody>
</table>

a) Use exponential regression to determine the annual rate of interest that Emily is paying on her loan.

\[y_1 = 3000(1.025)^x\]

\[x : \text{[0, 100, 10]}\]

\[y : \text{[0, 2, 0.5]}\]

\[n = 72 \text{ months}\]

b) Determine the sale price of the oven. How do you know?

\[\text{Initial cost} = \$3000\]

c) How much will Emily owe for the oven at the end of the 5th year?

\[y = 3000(1.025)^5 = \$3394.22\]
MULTIPLE CHOICE

6. Kevin purchased an antique Roman coin for $32 in 2006. He has been tracking the value of the coin every year since he bought it.

<table>
<thead>
<tr>
<th>Years since Purchase, t</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value ($), A(t)</td>
<td>32.00</td>
<td>38.56</td>
<td>48.00</td>
<td>53.76</td>
<td>67.25</td>
<td>80.64</td>
</tr>
</tbody>
</table>

a) Which regression equation best models the value of the coin over time?

A. \( A(t) = 9.5t + 29.4 \)  
B. \( A(t) = 32(1.2)^t \)  
C. \( A(t) = 0.9t^2 + 5t + 32.4 \)  
D. \( A(t) = 1.2(32)^t \)

b) Assume the same growth rate as in part a). What will the coin most likely be worth 10 years after it was purchased?

A. $120  
B. $150  
C. $200  
D. $320

WRITTEN RESPONSE

7. Arkita recently opened a restaurant. Her accountant told her that, for tax purposes, the depreciation rate of her restaurant equipment will be 15%, starting after the 2nd year.

a) At the end of Year 2, the value of the equipment is $40 000. Determine its value over the next 6 years. Explain your method.

<table>
<thead>
<tr>
<th>Time (end of year)</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value of Equipment ($)</td>
<td>40 000</td>
<td>$30 200</td>
<td>$28 900</td>
<td>$24 565</td>
<td>$20 980</td>
<td>$25 748.21</td>
</tr>
</tbody>
</table>

b) Use your values from part a) to create a scatter plot; then perform an exponential regression to determine a function that models this situation.

\[
Y = 58363.33(0.85)^x
\]

c) How much will Arkita’s equipment be worth 10 years after the purchase date? Explain.

\[
Y = 10 899.62 \approx $10 900 \text{ will be the value of her restaurant equipment.}
\]