5.4 Modelling Data with a Curve of Best Fit

Today's Focus: Determine the quadratic or cubic function that best fits a set of data, and use the function to solve a problem.

INVESTIGATION!!!

The top of a roller coaster is shown in the photograph below. Describe the characteristics of a polynomial function that might be used to model the shape of this section of the track.

KEY IDEAS!!!

- If the points on a scatter plot seem to follow a predictable curved pattern, then there may be a quadratic or cubic relationship between the independent variable and the dependent variable.

IMPORTANT THINGS TO REMEMBER!!!

- If the points on a scatter plot follow a quadratic or cubic trend, then graphing technology can be used to determine and graph the equation of the curve of best fit.
- To solve an equation, you can graph the corresponding function of each side of the equation. The $x$-coordinate of the point of intersection is the solution to the equation.
- Technology uses polynomial regression to determine the curve of best fit. Polynomial regression results in an equation of a curve that balances the points on both sides of the curve.
- A curve of best fit can be used to predict values that are not recorded or plotted. Predictions can be made by reading values from the curve of best fit on a scatter plot or by using the equation of the curve of best fit.
Example 1: The following graph shows a quadratic curve of best fit (in red) for the number of Canadian births from 1948 to 1968.

![Graph of Canadian Births](image)

Statistics Canada

a) Describe the trend in the data.

Increasing then decreasing \(\rightarrow\) curved

b) Based on the graph, in what year did the greatest number of births occur?

1960

c) How many births took place in 1965?

\(~ 420,000\)

d) In what years did more than 400,000 births occur?

1953–1965
Example 2: The data in the following scatter plot represents several trials of an experiment that was conducted by an athlete to compare his heart rate to his power output. The equation of the cubic regression function that models this data is

\[ H = -0.000\ 002\ 4x^3 + 0.001\ 930\ 2x^2 - 0.224\ 951\ 6x + 104.820\ 761\ 9 \]

where \( H \) represents the athlete’s heart rate in beats per minute (bpm) and \( x \) represents his power output in watts (W).

Heart Rate versus Power

<table>
<thead>
<tr>
<th>Power output (W)</th>
<th>Heart rate (bpm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>160</td>
<td>100</td>
</tr>
<tr>
<td>210</td>
<td>110</td>
</tr>
<tr>
<td>260</td>
<td>120</td>
</tr>
<tr>
<td>310</td>
<td>130</td>
</tr>
<tr>
<td>360</td>
<td>140</td>
</tr>
<tr>
<td>410</td>
<td>150</td>
</tr>
<tr>
<td>460</td>
<td>160</td>
</tr>
</tbody>
</table>

a) Use the regression equation to estimate the athlete’s heart rate when his power output is 310 W.

\[ x = 310 \quad H = -0.000\ 002\ 41(310)^3 + 0.001\ 932\ 02(310)^2 - 0.224\ 951\ (310) + 104.820\ 761\ 9 = 149 \]

b) Use the curve of best fit to estimate the athlete’s power output when his heart rate is 130 bpm.

\[ \sim 240\ W \]
Example 3: Mr. Tran hit a golf ball from the top of a hill. The height of the ball above the green is given in the table below.

<table>
<thead>
<tr>
<th>Time (s)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Height (m)</td>
<td>52.5</td>
<td>73.2</td>
<td>74.6</td>
<td>55.8</td>
<td>16.1</td>
</tr>
</tbody>
</table>

a) Describe the characteristics of the data.

- Increases then decreases → curved.

b) Determine the equation of the quadratic regression function that models the data.

\[ y = -10.071x^2 + 51.409x + 11 \]

c) Use your equation to determine the height of the ball at

- a. 0 sec
- b. 2.5 sec
- c. 4.5 sec

\[ H = 11 \text{ m} \]
\[ H = 76.575 \text{ m} \]
\[ H = 38.39 \text{ m} \]

d) When did the ball hit the ground?

\[ x = \text{time} = 5.31 \text{ s} \]