5.2 Characteristics of the Equations of Polynomial Functions

Keep in Mind

- When a polynomial function is in standard form
  - The maximum number of x-intercepts the graph may have is equal to the degree of the function.
  - The maximum number of turning points the graph may have is equal to one less than the degree of the function.
  - The degree and leading coefficient indicate the end behaviour of the graph of the function.
  - The y-intercept of the graph is equal to the constant term of the function.
- Linear and cubic polynomial functions have similar end behaviour.
  - Negative leading coefficient: the graph extends from Quadrant II to Quadrant IV.
  - Positive leading coefficient: the graph extends from Quadrant III to Quadrant I.
- Quadratic polynomial functions have a different end behaviour.
  - Negative leading coefficient: the graph extends from Quadrant III to Quadrant IV.
  - Positive leading coefficient: the graph extends from Quadrant II to Quadrant I.

Example 1

Determine the following characteristics of each function, using its equation.

a) \( f(x) = 4x + 2 \)

b) \( f(x) = -5x^2 + 2x - 1 \)

Solution

a) I considered each characteristic of the equation \( f(x) = 4x + 2 \).

- The value of the greatest exponent is 1, so the degree is 1. Since the degree is 1, the function is linear, so its graph is a line, and the graph has one x-intercept.
- The constant term is 2, so the y-intercept is 2.
- The leading coefficient, 4, is positive, so the graph extends from Quadrant III to Quadrant I.

TIP

In your descriptions of characteristics of a function include
- number of x-intercepts
- y-intercept
- end behaviour
- domain
- range
- number of possible turning points
• There are no restrictions or $x$. The domain is $\{x \mid x \in \mathbb{R}\}$.
• There are no restrictions or $y$. The range is $\{y \mid y \in \mathbb{R}\}$.
• This function is linear, so it has no turning points.

b) I considered each characteristic of the equation $f(x) = -5x^2 + 2x - 1$.

• The value of the greatest exponent is 2, so the degree is 2. Since the degree is 2, the function is quadratic, so its graph is a parabola, and the graph may have 0, 1, or 2 $x$-intercepts.
• The constant term is $-1$, so the $y$-intercept is $-1$.
• The leading coefficient, $-5$, is negative, and the equation is quadratic, so the graph extends from Quadrant III to Quadrant IV.
• There are no restrictions or $x$. The domain is $\{x \mid x \in \mathbb{R}\}$.
• The range is $\{y \mid y \leq \text{maximum}, y \in \mathbb{R}\}$.
• This function is quadratic, so it has one turning point.

Example 2

Match each graph to the correct polynomial function.

A. 

B. 

i) $f(x) = x^2 + 3x - 1$

ii) $g(x) = -x^2 + 3x + 2$

Solution

Step 1. I looked at the number of turning points in each graph.
Each graph has one turning point, so both $f(x)$ and $g(x)$ are quadratic functions.

Step 2. I looked at the end behaviour of each graph.
Graph A extends from Quadrant III to Quadrant IV, so the leading coefficient must be negative. Graph A matches with $g(x)$.
Graph B extends from Quadrant II to Quadrant I, so the leading coefficient must be positive. Graph B matches with $f(x)$.

Step 3. I verified my conclusion by looking at the $y$-intercepts.
The $y$-intercept of Graph A is 2. The constant term of $g(x)$ is 2, so again, the graph and equation match.
The $y$-intercept of Graph B is $-1$, matching the constant term of $f(x)$.
Graph A matches with $g(x)$, and Graph B matches with $f(x)$.
Practice

1. Match each graph with the correct polynomial function. Provide your reasoning.

**Graph A**

number of x-intercepts: 1

y-intercept: 2

end behaviour: from Quadrant II to Quadrant IV
so the leading coefficient is negative

domain: \( x \in \mathbb{R} \) range: \( y \in \mathbb{R} \)

number of turning points: 0

Graph A represents a linear polynomial function. It matches with function \( f(x) \)

**Graph B**

number of x-intercepts: 1

y-intercept: -1

end behaviour: from Quadrant II to Quadrant IV
so the leading coefficient is negative

domain: \( x \in \mathbb{R} \) range: \( y \in \mathbb{R} \)

number of turning points: 2

Graph B represents a cubic polynomial function. It matches with function \( g(x) \)

**Graph C**

number of x-intercepts: 1

y-intercept: -2

end behaviour: from Quadrant III to Quadrant I
so the leading coefficient is positive

domain: \( x \in \mathbb{R} \) range: \( y \in \mathbb{R} \)

number of turning points: 0

Graph C represents a cubic polynomial function. It matches with function \( g(x) \)

**Graph D**

number of x intercepts: 1

y-intercept: 2

end behaviour: from Quadrant III to Quadrant I
so the leading coefficient is positive

domain: \( x \in \mathbb{R} \) range: \( y \in \mathbb{R} \)

number of turning points: 0

Graph D represents a linear polynomial function. It matches with function \( j(x) \)
2. Write a polynomial function that satisfies each set of characteristics.
   a) extending from Quadrant III to Quadrant IV, one turning point, y-intercept of 4
   b) degree 1, decreasing, y-intercept of -2
   c) extending from Quadrant III to Quadrant I, y-intercept of -3
   d) two turning points, y-intercept of 5

NUMERICAL RESPONSE

3. State the characteristics of each polynomial function.
   a) \( f(x) = -3x^2 - 2x + 1 \)
      • degree: 2
      • leading coefficient: -3
      • constant term: 1
      • number of x-intercepts: 2 (see on graphing calc)
      • y-intercept: 1
      • extends from Quadrant III to Quadrant IV
      • domain: \( x \in \mathbb{R} \)
      • range: \( y \leq \text{max} \), \( y \leq \frac{3}{2} \)
      • number of turning points: 1

b) \( h(x) = 4x^3 + 2x^2 - x + 34 \)
   • degree: 3
   • leading coefficient: 4
   • constant term: 34
   • number of x-intercepts: 1
   • y-intercept: 34
   • extends from Quadrant III to Quadrant I
   • domain: \( x \in \mathbb{R} \)
   • range: \( y \leq \text{max} \)
   • number of turning points: 2

WRITTEN RESPONSE

4. The life expectancy of Canadian males born from 1920 to 2008 can be modelled by the polynomial function \( E(x) = 0.2339x - 390.8 \), where \( E \) is the life expectancy in years and \( x \) is the year of birth.
   a) Describe the characteristics of the graph of the polynomial function.
   Explain your answer.

b) Would you use this graph to estimate the life expectancy of a male born in the year 1000? Explain.

   No, that would yield a negative age. The age expectancy is only for the years 1920-2008.