3.6 Independent Events

Today's Focus: Understand and solve problems that involve independent events

INVESTIGATION

The Fortin family has two children. Cam determines the probability that the family has two girls. Emily determines the probability that the family has two girls, given that the first child is a girl. How are these probabilities similar and how are they different?

Cam
- 2 girls
- 1 girl & 1 boy
- 2 boys

Cam does not consider the order of (1 boy & 1 girl).

Emily

\[ P(G \cap G) = P(G) \cdot P(G | G) = \frac{1}{4} \]

Emily is factoring order in.

\[ \frac{GB}{BC} \quad \frac{GG}{BB} = \frac{1}{4} \]

The first event does not affect the second event.

KEY IDEAS!!!

- If the probability of event \( B \) does not depend on the probability of event \( A \) occurring, then these events are called \textit{independent events}. For example, tossing tails with a coin and drawing the ace of spades from a standard deck of 52 playing cards are independent events.
- The probability that two independent events, \( A \) and \( B \), will both occur is the product of their individual probabilities:

\[ P(A \cap B) = P(A) \cdot P(B) \]
IMPORTANT THINGS TO REMEMBER!!!

• A tree diagram is often useful for modelling problems that involve independent events.
• Drawing an item and then drawing another item, after replacing the first item, results in a pair of independent events.

Example 1: For each situation, classify the events as either independent or dependent. Justify your classification.
  a) A four-colour spinner is spun, and a die is rolled. The first event is spinning red, and the second event is rolling a 2.

  independent \rightarrow \text{rolls don't affect spins \& visa versa.}

  b) A red die and a green die are rolled. The first event is rolling a 1 on the red die, and the second event is rolling a 5 on the green die.

  independent \rightarrow \text{red die doesn't affect green die.}

  c) Two cards are drawn, \textbf{without being replaced}, from a standard deck of 52 playing cards. The first event is drawing a king, and the second event is drawing an ace.

  dependent \rightarrow "without replacement"

  d) There are 30 cards, numbered 1 to 30, in a box. Two cards are drawn, one at a time, \textbf{with replacement}. The first event is drawing a prime number, and the second event is drawing a number that is a multiple of 5.

  independent \rightarrow "with replacement"
Example 2: Anne and Chantelle are playing a die and coin game. Each turn consists of rolling a regular die and tossing a coin. Points are awarded for rolling a 6 on the die and/or tossing heads with the coin:
- 1 point for either outcome
- 3 points for both outcomes
- 0 points for neither outcome

Players alternate turns. The first player who gets 10 points wins.

a) Determine the probability that Anne will get 1, 3, or 0 points on her first turn.

b) Verify your results for part a). Explain what you did.

\[ P(3 \text{ points}) = \frac{1}{12} \]

\[ P(1 \text{ point}) = \frac{1}{12} + \frac{5}{12} = \frac{6}{12} = \frac{1}{2} \]

\[ P(0 \text{ points}) = \frac{5}{12} \]

b) They all should add to 1. 

\[ P(3 \text{ or } 1 \text{ or } 0) = \frac{1}{12} + \frac{1}{2} + \frac{5}{12} = \frac{12}{12} = 1 \]
Example 3: All 1000 tickets for a charity raffle have been sold and placed in a drum. There will be two draws. The first draw will be for the grand prize, and the second draw will be for the consolation prize. After each draw, the winning ticket will be returned to the drum so that it might be drawn again. Max has bought five tickets. Determine the probability, to a tenth of a percent, that he will win at least one prize.

\[
P(W\text{ at least 1}) = \frac{1}{40 000} + \frac{199}{40 000} + \frac{199}{40 000} = \frac{399}{40 000}
\]

or

\[
0.009975 \approx 0.9975\%
\]