3.6 Independent Events

Keep in Mind

- Events are independent when the probability of one does not depend on the probability of the other. For example, drawing a heart from a standard deck of 52 playing cards, replacing the card, and then drawing another heart from the same deck are independent events.
- Drawing an item and then drawing another item, after replacing the first item, results in a pair of independent events.
- The probability that two independent events, A and B, will both occur is the product of their individual probabilities:
  \[ P(A \cap B) = P(A) \cdot P(B) \]
- A tree diagram is often useful for modelling problems with independent events.

Example

Alin rolls two 4-sided dice and tosses one coin. Determine the probability that the product of the dice is an odd number and the coin lands tails.

Solution

Step 1. I defined the two events and considered whether they were dependent or independent.

Let O represent rolling an odd product.

Let T represent tossing tails with the coin.

Since these events do not depend on each other, they are independent.

Step 2. I already knew \( P(T) = \frac{1}{2} \). I determined \( P(O) \) using a table.

<table>
<thead>
<tr>
<th>Product of Two 4-Sided Dice</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \cdot )</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td>3</td>
</tr>
<tr>
<td>4</td>
</tr>
</tbody>
</table>

There are 16 possible products, of which 4 are odd.

\[ P(O) = \frac{4}{16}, \text{ or } \frac{1}{4} \]

Step 3. I multiplied to determine the probability that both events will occur.

\[ P(O \cap T) = P(O) \cdot P(T) \]

\[ P(O \cap T) = \frac{1}{4} \cdot \frac{1}{2} \]

\[ P(O \cap T) = \frac{1}{8} \]

There is a \( \frac{1}{8} \) chance that the product will be odd and the coin will land tails.
Practice

1. Classify the events in each situation as independent or dependent.

a) A spinner with 5 equal coloured sections—red, blue, green, yellow, and white—is spun twice in a row. The first event is spinning red, and the second event is also spinning red.

   independent

b) Two marbles are drawn, without being replaced, from a bag containing 7 red marbles and 3 blue marbles. The first event is drawing a red marble, and the second event is drawing a blue marble.

dependent

c) A coin is tossed and a standard die is rolled. The first event is tossing tails, and the second event is rolling a 3 on the die.

independent

d) There are 12 cards, one for each month, in a box. A card is drawn from the box and replaced, then a second card is drawn. The first event is drawing a month with only 4 letters, and the second event is drawing a month that contains an R.

independent

2. For each situation described in question 1, determine the probability that both events will occur.

a) \( P(RNR) = \frac{1}{5} \cdot \frac{1}{5} = \frac{1}{25} \)  

b) \( P(RNB) = \frac{7}{10} \cdot \frac{3}{9} = \frac{21}{90} = \frac{7}{30} \)  

c) \( P(TNB) = \frac{1}{2} \cdot \frac{1}{6} = \frac{1}{12} \)  

d) \( P(4NR) = \frac{2}{12} \cdot \frac{3}{12} = \frac{16}{144} = \frac{2}{18} \)

3. Marcel works two shifts a week at a coffee shop, one on a weekday and one on the weekend. He is available during the week from Tuesday to Thursday and on both days on the weekend. His boss randomly chooses which shifts Marcel will work.

a) Does choosing the two shifts for one week involve dependent or independent events?

   independent

b) Determine the probability that Marcel will work on Tuesday and Saturday.

\[ P(TNS) = P(T) \cdot P(S) = \frac{1}{3} \cdot \frac{1}{2} = \frac{1}{6} \]
4. Martina also works at the coffee shop. She is available to work two shifts from Monday to Thursday.
   a) Are the two shifts dependent or independent events?
   
   
   dependent
   
   b) Determine the probability that Martina will work on Monday and Tuesday.
   
   \[ P(\text{M} \cap \text{T}) = \frac{1}{4} \cdot \frac{1}{2} = \frac{1}{8} \]

5. Two standard dice, one red and one green, are rolled. Harris scores a point if an odd number is rolled on the red die and a point if a multiple of 3 is rolled on the green die.
   a) Draw a tree diagram that shows all the possible outcomes.
   
   
   b) Determine the probability of Harris scoring 2 points.
   
   \[ P(\text{odd} \cap \text{multiple of 3}) = \frac{1}{2} \cdot \frac{1}{3} = \frac{1}{6} \]
   
   c) Determine the probability of Harris scoring 1 point.
   
   \[ P(\text{odd} \cap \text{not multiple of 3}) = \frac{1}{2} \cdot \frac{2}{3} = \frac{1}{3} \]

6. Design a spinner so that when you roll a standard die and spin the spinner, the probability of rolling a multiple of 3 and spinning red is \( \frac{1}{24} \).
   
   \[ \frac{1}{3} \cdot \frac{1}{8} = \frac{1}{24} \]  
   
   MULTIPLE CHOICE

**Questions 7 to 9 refer to the following scenario.**

Ronnie is going on a cruise off the coast of Newfoundland. The travel brochure says the probability of seeing a whale is \( \frac{9}{10} \) and the probability of seeing a puffin is \( \frac{3}{5} \).

7. What is the probability that Ronnie will see a whale and a puffin?
   
   A. 0.30  
   B. 0.46  
   C. 0.54  
   D. 1.50

   C. 0.54

8. What is the probability that Ronnie will not see either sight?
   
   A. 0.04  
   B. 0.08  
   C. 0.40  
   D. 0.30

   A. 0.04

\[ \frac{1}{10} \cdot \frac{2}{5} = \frac{2}{50} = 0.04 \]
9. What is the probability that Ronnie will see only one of these sights?
   A. 0.30    B. 0.42    C. 0.50    D. 0.54

Questions 10 and 11 refer to the following scenario.
A 4-sided die, numbered 1 to 4, and an 8-sided die, numbered 1 to 8, are rolled.

10. What is the probability that the number on the 8-sided die is double the number on the 4-sided die?
   A. 0    B. \( \frac{1}{4} \)    C. \( \frac{1}{8} \)    D. \( \frac{1}{16} \)

11. What is the probability that the sum is odd?
   A. \( \frac{1}{2} \)    B. \( \frac{1}{4} \)    C. \( \frac{1}{8} \)    D. \( \frac{1}{16} \)

NUMERICAL RESPONSE

12. Two 8-sided dice, numbered 1 to 8, are rolled.
   a) The probability of rolling a sum of 12 is \( \frac{5}{64} \).
   b) The probability of rolling a sum of 1 is 0.
   c) The probability of rolling doubles is \( \frac{8}{64} = \frac{1}{8} \).

13. A paper bag contains 5 prize tickets for a movie pass, 10 prize tickets for an album download, and 15 prize tickets for a keychain. Suppose that a ticket is randomly drawn from the bag and replaced, and then a ticket is drawn again from the bag. You win the prize only if the same type of ticket is drawn both times.
   a) The probability of winning a movie pass is \( \frac{1}{36} \).
   b) The probability of winning any prize is \( \frac{7}{18} \).
   c) The probability of not winning a prize is \( \frac{11}{18} \).