3.5 Conditional Probability

Today’s Focus: Understand and solve problems that involve dependent events

INVESTIGATION

Jackie plays on a volleyball team called the Giants. The Giants are in a round-robin tournament with five other teams. The teams that they will play against will be selected at random. Determine the probability that their first game will be against the Clippers and their second game will be against the Maroons.

\[
\frac{1 \cdot \frac{1}{3} \cdot \frac{2}{5} \cdot \frac{1}{4}}{\text{Clippers} \times \text{Maroons} \times \text{Any} \times \text{Any} \times \text{Any}} = 6
\]

Total games = \( \frac{5!}{4! \cdot 1!} = 5 \times 4 = 120 \)

\[
P(1\text{st Clipper, 2nd Maroon}) = \frac{6}{120} = \frac{1}{20}
\]

OR

\[
P(1\text{st Clipper, 2nd Maroon}) = \frac{1 \cdot \frac{1}{4}}{\text{Clippers} \times \text{Maroons} \times \text{First} \times \text{Second} \times \text{4 teams left}}
\]

\[
P(1\text{st Clipper, 2nd Maroon}) = \frac{1}{20}
\]

KEY IDEAS!!!

- If the probability of one event depends on the probability of another event, then these events are called **dependent events**. For example, drawing a heart from a standard deck of 52 playing cards and then drawing another heart from the same deck without replacing the first card are dependent events.
- If event \( B \) depends on event \( A \) occurring, then the **conditional probability** that event \( B \) will occur, **given** that event \( A \) has occurred, can be represented as follows:

\[
P(B \mid A) = \frac{P(A \cap B)}{P(A)}
\]

\[
P(A \cap B) = P(A) \cdot P(B \mid A)
\]

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IMPORTANT THINGS TO REMEMBER!!

- If event $B$ depends on event $A$ occurring, then the probability that both events will occur can be represented as follows:
  \[ P(A \cap B) = P(A) \cdot P(B \mid A) \]
- A tree diagram is often useful for modelling problems that involve dependent events.
- Drawing an item and then drawing another item, without replacing the first item, results in a pair of dependent events.

**Example 1:** Valeria draws a card from a well-shuffled standard deck of 52 playing cards. Then she draws another card from the deck without replacing the first card. Are these two events dependent or independent?
  a) Are these two events dependent or independent? Dependent
  b) Determine the probability that both cards are diamonds.

  \[
P(A \cap B) = P(A) \cdot P(B \mid A)
  \]

  \[
P(\text{Diamonds} \cap \text{Diamonds}) = P(\text{Diamond 1st}) \cdot P(\text{Diamond 2nd} \mid \text{Diamond 1st})
  \]

  \[
  = \frac{13}{52} \cdot \frac{12}{51}
  \]

  \[
  = \frac{1}{17} \text{ or } 0.0588
  \]

**Example 2:** Valeria draws a card from a well-shuffled standard deck of 52 playing cards. Then she puts the card back in the deck, shuffles again, and draws another card from the deck. Are these two events dependent or independent? Independent

  a) Are these two events dependent or independent? Independent
  b) Determine the probability that both cards are diamonds.

  \[
P(A \cap B) = P(A) \cdot P(B)
  \]

  \[
  = \frac{13}{52} \cdot \frac{13}{52}
  \]

  \[
  = \frac{1}{16} \text{ or } 0.0625
  \]
Example 3: Nathan asks Riel to choose a number between 1 and 40 and then say one fact about the number. Riel says that the number he chose is a multiple of 4. Determine the probability that the number is also a multiple of 6.

\[ A = \{4, 8, 12, 16, 20, 24, 28, 32, 36, 40\} \]
\[ B = \{6, 12, 18, 24, 30, 36\} \]

\[
P(A \cap B) = \frac{3}{40} \quad P(A) = \frac{10}{40}
\]

\[
P(A \cap B) = P(A) \cdot P(B | A)
\]

\[
\frac{3}{40} = \frac{10}{40} \cdot P(B | A)
\]

\[
\frac{10}{40} = \frac{3}{10} = P(B | A)
\]

Example 4: According to a survey, 91% of Canadians own a cell phone. Of these people, 42% have a Smartphone. Determine, to the nearest percent, the probability that any Canadian you met during the month in which the survey was conducted would have a Smartphone.

Let \( C = \text{cell phone} \)
\( S = \text{smartphone} \)

\[
P(C) = 0.91
\]

\[
P(S | C) = 0.42
\]

\[
P(C \cap S) = P(C) \cdot P(S | C)
\]

\[
= (0.91) \cdot (0.42)
\]

\[
= 0.3822
\]

The probability that a Canadian has a Smartphone is 38.22%.

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