3.4 Mutually Exclusive Events

Today's Focus: Understand and solve problems that involve mutually exclusive and non-mutually exclusive events

INVESTIGATION

Carlos drew a single card from a standard deck of 52 playing cards. What is the probability that the card he drew is either an 8 or a black card?

\[
\text{favourable} = n(8) + n(\text{black}) - n(8 \cap \text{black})
\]
\[
= 4 + 26 - 2
\]
\[
= 28
\]
\[
P(8 \text{ or black}) = \frac{28}{52} = \frac{7}{13}
\]

KEY IDEAS!!!

- You can represent the favourable outcomes of two mutually exclusive events, A and B, as two disjoint sets.
  
  You can represent the probability that either A or B will occur by the following formula:
  
  \[
P(A \cup B) = P(A) + P(B)
  \]

- You can represent the favourable outcomes of two non-mutually exclusive events, A and B, as two intersecting sets.
  
  You can represent the probability that either A or B will occur by this formula:
  
  \[
P(A \cup B) = P(A) + P(B) - P(A \cap B)
  \]

An alternative formula is

\[
P(A \cup B) = P(A \setminus B) + P(B \setminus A) + P(A \cap B)
\]

When the two events are mutually exclusive, both formulas are equivalent, since

\[
n(A \cap B) = 0
\]

which results in

\[
P(A \cap B) = 0
\]

13

"Principles of Mathematics 12" Nelson Education
IMPORTANT THINGS TO REMEMBER!!

- You can use the Principle of Inclusion and Exclusion, which is used to count the elements in the union of two sets, to determine the probability of non-mutually exclusive events.

Example 1: Zach is playing a board game. He must roll two four-sided dice, numbered 1 to 4. He can move if he rolls a sum of 2 or a sum of 8.

\[ A = \{ (1,1), (1,3), (3,1), (3,3), (4,4) \} \]
\[ B = \{ (4,1), (4,4) \} \]

a) Use \( A \) and \( B \) to represent the two events that will allow Zach to move. Then draw a Venn diagram to illustrate \( A \) and \( B \).

b) Are \( A \) and \( B \) mutually exclusive or not mutually exclusive?

c) Determine the probability that Zach will roll a sum of 2 or a sum of 8.

d) Determine the probability that Zach will roll doubles or a sum of 6.

\[
P(\text{sum of 6}) = \frac{3}{16}
\]

\[
P(\text{doubles and sum of 6}) = \frac{1}{16}
\]

\[
P(\text{doubles or sum of 6}) = P(\text{doubles}) + P(\text{sum of 6}) - P(\text{doubles and sum of 6})
\]

\[
= \frac{4}{16} + \frac{3}{16} - \frac{1}{16} = \frac{6}{16} = \frac{3}{8}
\]
Example 2: Pearl is about to draw a card at random from a standard deck of 52 playing cards. If she draws a face card or a spade, she will win a point.

a) Draw a Venn diagram to represent the two events.

b) Are the events mutually exclusive?

c) Determine the probability of drawing a face card or a spade.

\[
\begin{align*}
n(F) &= 12 \
 n(F \cap S) &= 3 \
 n(F/S) &= 12 - 3 = 9 \\
n(S) &= 13 \
 n(S/F) &= 10 \
 n(U) &= 52
\end{align*}
\]

\[
P(F \cup S) = P(F) + P(S) - P(F \cap S) = \frac{12}{52} + \frac{13}{52} - \frac{3}{52} = \frac{22}{52} = \frac{11}{26}
\]

or

\[
P(F \cup S) = P(F/S) + P(S/F) + P(F \cap S) = \frac{9}{52} + \frac{10}{52} + \frac{3}{52} = \frac{22}{52} = \frac{11}{26}
\]

Example 3: The probability that Maria will go to the gym on Saturday is 0.75. The probability that she will go shopping on Saturday is 0.4. The probability that she will do neither is 0.2.

a) Draw a Venn diagram to represent the two events.

b) Are the two events mutually exclusive?

c) Determine the probability that Maria will do at least one of these activities on Saturday.

\[
P(G) + P(S) + P((G \cup S)') = 0.75 + 0.4 + 0.2 = 1.35
\]

\[
P(G \cap S) = 1.35 - 1 = 0.35
\]

\[
P(G/S) = 0.75 - 0.35 = 0.40
\]

\[
P(S/G) = 0.4 - 0.35 = 0.05
\]

\[
P(GUS) = 0.4 + 0.35 + 0.05 = 0.80
\]

"Principles of Mathematics 12" Nelson Education
Example 4: The following Venn diagram shows the declared population of Métis in Canada, where

\[ A = \{\text{Métis in Alberta and British Columbia}\}, \]
\[ M = \{\text{Métis in Manitoba and Saskatchewan}\}, \text{ and} \]
\[ C = \{\text{Métis in Canada}\}. \]

\[
\text{Total} = 144,945 + 119,920 + 124,180 = 389,045
\]

a) Determine the probability that a person who is Métis lives in Alberta or British Columbia.

\[
\frac{144,945}{389,045}
\]

b) Determine the probability that a person who is Métis lives in Manitoba or Saskatchewan.

\[
\frac{119,920}{389,045}
\]

c) Does \( P(A \cup M) = P(A) + P(M) \) in this situation? Explain.

Yes. They are mutually exclusive.

d) Determine the odds in favour of a person who is Métis living in one of the four Western provinces.

\# who live in all four = 144,945 + 119,920 = 264,865
\# who don't live in all four = 124,180

\[
264,865 : 124,180
\]

Reduced 2

\[
52,973 : 24,836
\]