Step 2. I used these cases to write down and evaluate an expression for the number of ways to pick 6 children with at least one boy and at least one girl.
\[
\gamma C_2 \cdot \gamma C_5 + \gamma C_2 \cdot \gamma C_4 + \gamma C_3 \cdot \gamma C_3 + \gamma C_4 \cdot \gamma C_2 + \gamma C_5 \cdot \gamma C_1 \\
= 882 + 2646 + 2940 + 1260 + 189, \text{ or } 7917 \text{ ways}
\]

Solution 2

Step 1. I chose an indirect reasoning strategy. I decided to determine
- the number of ways to choose any 6 children from 16,
- the number of ways to choose 6 boys from 7, and
- the number of ways to choose 6 girls from 9.

Step 2. I used my reasoning to write down and evaluate an expression for the number of ways to pick 6 children with at least one boy and at least one girl.
\[
\binom{16}{6} - \binom{7}{6} - \binom{9}{6} = 8008 - 7 - 84, \text{ or } 7917 \text{ ways}
\]

Practice

1. a) List all of the permutations of 3 symbols chosen from the symbols
\[
\bigcirc, \Box, \Diamond, \text{ and } \triangle.
\]
\[
\begin{array}{cccc}
CSD & CST & CDT & SDT \\
CDS & CTS & CTD & STD \\
SDC & SCT & DCT & DST \\
SCD & STC & DTC & DTS \\
DCS & TCS & TCD & TSD \\
DSC & TSC & TDC & TDS
\end{array}
\]

[24 permutations]

b) List all of the combinations of 3 symbols chosen from the same list.

- CSD, CST, CDT, SDT → 4 combinations

b) How is the number of permutations of 3 symbols related to the number of combinations of 3 symbols? Explain.

- 6 times more permutations (3! ways of arranging the subsets)

2. Evaluate the following.

a) \(5 C_2 = \frac{5!}{3! \cdot 2!}\), or \(10\)

c) \(10 C_5 = \frac{10!}{5! \cdot 5!}\), or \(252\)

b) \(\binom{7}{3} = \frac{7!}{3! \cdot 4!}\), or \(35\)

d) \(\binom{13}{1} = \frac{13!}{1! \cdot 12!}\), or \(13\)
3. a) In how many ways can a set of 6 cards be dealt from just the hearts in a standard deck?

\[ \binom{13}{6} = 1,716 \text{ ways} \]

b) In how many ways can a set of 6 cards be dealt from just the hearts in a standard deck if the set of 6 cards includes no face cards?

\[ \binom{10}{6} = 210 \text{ combinations} \]

4. A committee is to be chosen from a set of 11 students, of whom Tomas is one. The committee has 1 chair and 4 other members. In how many ways can the committee be selected?

a) if Tomas is the chair?

\[ \binom{1}{1} \times \binom{10}{4} = 210 \text{ ways} \]

b) if Tomas is on the committee, but someone else is the chair?

\[ \binom{10}{1} \times \binom{9}{3} \times \binom{1}{1} = 840 \text{ ways} \]

\[ \binom{10}{1} \times \binom{9}{3} \times \frac{10}{C_1} = 840 \text{ ways} \]

\[ \binom{10}{1} \times \binom{9}{3} \times \frac{10}{C_1} = 840 \text{ ways} \]

5. a) Write an expression for \(\binom{n}{n-r}\) in terms of factorials.

\[ \frac{n!}{(n-(n-r))! \cdot (n-r)!} = \frac{n!}{r! \cdot (n-r)!} \]

b) By simplifying your expression, show that \(\binom{n}{n-r} = \binom{n}{r}\).

\(\binom{n}{n-r} = \frac{n!}{r! \cdot (n-r)!}\) as seen from above

\(\binom{n}{r} = \frac{n!}{r! \cdot (n-r)!}\) with formula

6. An improv team has spots for 3 women and 2 men, and 14 women and 8 men audition. How many different combinations of people could be chosen for the team?

7. TIP

Decide whether a problem involves the Fundamental Counting Principle or mutually exclusive cases.
7. Suppose you are choosing three of the symbols +, −, ×, ÷, and √.

a) Write an expression in the form \( _nC_r \) for the total number of combinations.

\[ 5C_3 = \frac{5!}{2!3!} = 10 \text{ combinations} \]

b) You decide to break this into two cases:

- choose 2 symbols from +, −, ×, and ÷; and also choose the symbol √,
  or
  \[ 4C_2 \cdot 1C_1 \]

- choose 3 symbols from +, −, ×, and ÷. Write expressions in the same form for the total number of combinations for the two cases.

\[ 4C_3 \]

c) Complete these equations relating your expressions from parts a) and b).

\[ 5C_3 = 4C_2 + 4C_3 = 6 + 4 = 10 \]

d) The start of Pascal's triangle is shown on the right. Circle the numbers in the triangle that correspond to your equation from part c).

Pascal's triangle

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
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<tbody>
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<tr>
<td>1</td>
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<tr>
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<tr>
<td>1</td>
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<td>6</td>
</tr>
<tr>
<td></td>
<td></td>
<td>10</td>
</tr>
</tbody>
</table>

8. Solve each equation. State any restrictions on the variable.

a) \( _nC_2 = 21 \)

\[ \frac{n!}{(n-2)2!} = 21 \]

\[ \frac{n(n-1)(n-2)!}{(n-2)!2!} = 21 \times 2 \]

\[ n^2 - n = 42 \]

\[ n^2 - n - 42 = 0 \]

\[ n = 7, -6 \]

\[ n = 7 \]  

\[ n = 7 - 6 \]

MULTIPLE CHOICE

9. Roger is counting the number of 5-card hands that can be dealt from a standard deck and contain at least 2 black cards. Which of these expressions applies?

A. \( \binom{26}{5} + \binom{26}{1} \binom{26}{4} \)

B. \( \binom{26}{0} \binom{26}{5} + \binom{26}{1} \binom{26}{4} + \binom{26}{2} \binom{26}{3} \)

C. both expressions

D. neither expression

WRITTEN RESPONSE

10. How many ways are there to divide a committee of 12 students into

a) 3 groups of 4?

\[ \binom{12}{4} \times \binom{8}{4} \times \binom{4}{4} \]

34 650 ways

b) 4 groups of 3?

\[ \binom{12}{3} \binom{9}{3} \binom{6}{3} \binom{3}{3} = \frac{220 \times 84 \times 20 \times 1}{4!} \]

369 600 ways

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\[ \text{b)} \quad \frac{(n+1)!}{(n+1-3)!3!} = 4 \cdot \frac{n!}{(n-2)!2!} \]

\[ \frac{(n+1)!}{(n-2)!3!} = 4 \frac{n!}{(n-2)!2!} \]

\[ \frac{(n+1)n(n-1)(n-2)!}{(n-2)!3!} = 4 \frac{n(n-1)(n-2)!}{(n-2)!2!} \]

\[ \frac{(n+1)n(n-1)}{6} = \frac{4n(n-1)}{2} \]

\[ \frac{2(n+1)n(n-1)}{(n+1)(n-1)} = \frac{24n(n-1)}{n(n-1)} \]

\[ 2(n+1) = 24 \]

\[ 2n + 2 = 24 \]

\[ -2 \quad -2 \]

\[ \frac{2n}{2} = 2 \]

\[ n = 11 \]

c) \[ \begin{align*}
8C_0 & \rightarrow 0 \\
8C_1 & \rightarrow 8 \\
8C_2 & \rightarrow 28 \\
8C_3 & \rightarrow 56 \\
8C_4 & \rightarrow 70 \\
8C_5 & \rightarrow 56 \\
8C_6 & \rightarrow 28 \\
8C_7 & \rightarrow 8 \\
8C_8 & \rightarrow 1
\end{align*} \]

\[ r = 2 \text{ or } 6 \]