Relations and Functions

Graphing Calculator Window Format
\[ x: [x_{\text{min}}, x_{\text{max}}, x_{\text{sc}}] \]
\[ y: [y_{\text{min}}, y_{\text{max}}, y_{\text{sc}}] \]

Exponents and Logarithms
\[ y = a^x \iff x = \log_a y \]
\[ \log_b c = \frac{\log_c b}{\log_c a} \]

Laws of Logarithms
\[ \log_b (M \cdot N) = \log_b M + \log_b N \]
\[ \log_b \left( \frac{M}{N} \right) = \log_b M - \log_b N \]
\[ \log_b (M^n) = n \log_b M \]

Exponential functions
\[ y = a \cdot b^x \]

Sinusoidal functions
\[ y = a \cdot \sin(bx + c) + d \]
Period = \( \frac{2\pi}{b} \)

Quadratic equations
For \( ax^2 + bx + c = 0 \)
\[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]

Probability
\[ n! = n(n-1)(n-2)\ldots3\cdot2\cdot1, \]
where \( n \in \mathbb{N} \) and \( 0! = 1 \)
\[ P_r = \frac{n!}{(n-r)!} \]
\[ C_r = \frac{n!}{(n-r)!r!} \]

Logical Reasoning
\[ A' \] Complement
\[ \emptyset \] Empty set
\[ \cap \] Intersection
\[ \subseteq \] Subset
\[ \cup \] Union
Diploma Exam Specs & Design

Each Mathematics 30-2 Diploma Examination is designed to reflect the core content outlined in the Mathematics 30–2 Program of Studies. The examination is limited to those outcomes that can be measured by a machine-scored paper-and-pencil test. Therefore, the percentage weightings shown below will not necessarily match the percentage of class time devoted to each topic. The diploma examination was developed to be completed in 2.5 hours; however, an additional 0.5 hour is allowed for students to complete the exam.

<table>
<thead>
<tr>
<th>Question Format</th>
<th>Number of Questions</th>
<th>Emphasis</th>
</tr>
</thead>
<tbody>
<tr>
<td>Multiple Choice</td>
<td>28</td>
<td>70%</td>
</tr>
<tr>
<td>Numerical Response</td>
<td>12</td>
<td>30%</td>
</tr>
</tbody>
</table>

Conceptual, procedural, and problem-solving cognitive levels are addressed throughout the examination. The approximate emphasis of each cognitive level is given below:

**Multiple Choice and Numerical Response**

- Conceptual → Theoretical: 34%
- Procedural → Routine Math: 30%
- Problem Solving → Word Problems: 36%

**Diploma Examination Content**

- Logic & Reasoning: 17%
- Probability: 33%
- Relations and Functions: 50%
- Research Project: 0%
SET THEORY & LOGIC REASONING

Specific Outcome 1
Analyze puzzles and games that involve numerical and logical reasoning, using problem-solving strategies.

EXAMPLE 1

Use the following information to answer the next question
A pattern of pictures is shown below. The first picture is the original. In each subsequent picture, each shaded square has stayed in the same place or moved to a square horizontally, vertically, or diagonally adjacent to its previous location. The shaded square undergoes the same movement in each subsequent step.

Determine the next picture in the pattern

EXAMPLE 2

Use the following information to answer the next question

Three rows of a pattern are shown below.
Row 1  \(1 \times 8 + 1 = 9\)
Row 2  \(12 \times 8 + 2 = 98\)
Row 3  \(123 \times 8 + 3 = 987\)

The fifth row of the pattern will be
Row 4  \(1234 \times 8 + 4 = 9876\)
Row 5  \(12345 \times 8 + 5 = 98765\)
EXAMPLE 3

Use the following information to answer the next question

The diagram below shows a 4 by 4 KenKen puzzle. Here are the rules:
- The numbers you can use in a puzzle depend on the size of the grid. If it’s a 3 x 3 grid, you’ll use the numbers 1–3. In a 4 x 4 grid, use numbers 1–4.
- The heavily-outlined groups of squares in each grid are called “cages.” In the upper-left corner of each cage, there is a “target number” and a math operation (+, −, x, ÷).
- Fill in each square of a cage with a number. The numbers in a cage must combine—in any order, using only that cage’s math operation—to form that cage’s target number.

John is about to start the puzzle.

\[
\begin{align*}
1 \times 4 \times 3 & = 12 \\
1 \times 3 \times 4 & = 12 \\
3 \times 1 \times 4 & = 12 \\
3 \times 4 \times 1 & = 12 \\
4 \times 1 \times 3 & = 12 \\
4 \times 3 \times 1 & = 12 \\
\end{align*}
\]

In how many different ways can John fill in the shaded section?

Specific Outcome 2
Solve problems that involve the application of set theory.

EXAMPLE 4
There are 28 students on the school track and field team.
- 19 have blonde hair.
- 8 have green eyes.
- 9 do not have blonde hair or green eyes.

**A.** How many students have blonde hair and green eyes?

\[
19 - 8 = \text{?}
\]

**B.** How many students have green eyes but not blonde hair?

\[
19 + 8 = 27 - 19 = 8 \text{ overlap}
\]
**EXAMPLE 5**

Which of the following rows includes two groups that would be an example of disjoint sets?

<table>
<thead>
<tr>
<th>Row</th>
<th>Group 1</th>
<th>Group 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>A.</td>
<td>People who regularly drink coffee</td>
<td>People who regularly drink tea</td>
</tr>
<tr>
<td>B.</td>
<td>People who have a home phone line</td>
<td>People who have a cellular phone line</td>
</tr>
<tr>
<td>C.</td>
<td>The set of all prime numbers ( 2 \text{ to } 20 )</td>
<td>The set of all even numbers</td>
</tr>
<tr>
<td>D.</td>
<td>The set of all multiples of 5 ( 5 \text{ to } 20 )</td>
<td>The set of all factors of 24 ( 1, 2, 3, 4, 6, 8, 12, 24 )</td>
</tr>
</tbody>
</table>

**EXAMPLE 6**

Use the following information to answer the next three questions:

The universal set of the primary and secondary colours in art contains the colors yellow, blue, red, orange, green, purple, and black, as shown below.

If \( R \) is the set of colors that contain red, then all of the colors in the complement of \( R \), \( R' \), are:

\[
R' = \{ \text{Yellow, Green, Blue} \}
\]
EXAMPLE 7

Use the following information to answer the next question

Amber used a Venn diagram generator to compare the letters that are common to the words APPLES, BANANAS and PEARS. The generator created the following diagram:

In the circle labeled APPLES, what does the “1” refer to?

The letter L is only in Apples and not in Bananas or Pears.

EXAMPLE 8

Use the following information to answer the next question

There are 35 students in John’s homeroom class. There are 5 students who take English and biology, and 7 students who take neither of these subjects. There are 3 more students taking only English than there are students taking only biology.

The number of students in John’s homeroom who take biology only is

10 students will only take Bio
EXAMPLE 9

Grade 12 students at a high school were required to take at least one of physics, chemistry, or biology.

- 37 took physics.
- 62 took chemistry.
- 68 took biology.
- 27 took physics and chemistry.
- 15 took physics and biology.
- 33 took chemistry and biology.
- 12 students took all three sciences.

How many students were in Grade 12 that year?

\[32 + 14 + 7 + 15 + 12 + 3 + 21 = 104\]

EXAMPLE 10 – Principle of Exclusion and Inclusion

Use the following clues to answer the questions below:

- 28 children have a dog, a cat, or a bird.
- 13 children have a dog.
- 13 children have a cat.
- 13 children have a bird.
- 4 children have only a dog and a cat.
- 3 children have only a dog and a bird.
- 2 children have only a cat and a bird.
- No child has two of each type of pet.

a) How many children have a cat, a dog, and a bird?

b) How many children have only one pet?

\[
\begin{align*}
D & = 3 + 4 + 5 - (x + 4) - (x + 2) - (x + 3) + x \\
C & = 3 + 4 - x - 2 - x + 2 + x - 3 + x \\
B & = 3 + 0 - 2x \\
1 & = x
\end{align*}
\]

a) I have a cat, dog & bird.

b) 7+5+6 = 18 have only 1 pet.
1. For an assignment, Sandra created several 3 by 3 magic squares. Which magic square below is not correct?

A.  
\[
\begin{array}{ccc}
2 & 7 & 6 \\
9 & 5 & 1 \\
4 & 3 & 8 \\
\end{array}
\]

B.  
\[
\begin{array}{ccc}
8 & 1 & 6 \\
3 & 5 & 7 \\
4 & 9 & 2 \\
\end{array}
\]

C.  
\[
\begin{array}{ccc}
6 & 1 & 8 \\
7 & 5 & 3 \\
2 & 9 & 4 \\
\end{array}
\]

D.  
\[
\begin{array}{ccc}
4 & 9 & 2 \\
1 & 6 & 8 \\
7 & 3 & 5 \\
\end{array}
\]

Use the following information to answer the next question

Consider the series:

\[
\begin{array}{c}
\ast & B \\
\ast & B \\
\ast & B \\
\ast & B \\
\end{array}
\]

2. The diagram that best completes the series is

A.  
\[
\begin{array}{c}
\ast \\
B \\
\end{array}
\]

B.  
\[
\begin{array}{c}
\ast \\
B \\
\end{array}
\]

D.  
\[
\begin{array}{c}
\ast \\
B \\
\end{array}
\]

C.  
\[
\begin{array}{c}
\ast \\
B1 \\
\end{array}
\]
Use the following information to answer the next question:

Pedro is playing a game of Sudoku,

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td></td>
<td></td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>7</td>
<td>9</td>
<td>4</td>
</tr>
<tr>
<td>1</td>
<td>9</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>5</td>
<td></td>
<td></td>
<td>7</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>9</td>
<td>B</td>
</tr>
<tr>
<td>7</td>
<td>C</td>
<td></td>
<td>3</td>
</tr>
</tbody>
</table>

3. In which of the following squares should he put the number 1?
   A. A
   B. B
   C. C
   D. D

4. Of the grade 12 students in Cora’s high school, 60% take Math 30-2, 20% take both Math 30-2 and Science 30, and 25% take neither. What percentage of grade 12 students take Science 30 only?
   A. 5%
   B. 15%
   C. 35%
   D. 75%

Use the following information to answer the next question:

5. Given the Venn diagram, which statement correctly describes the shaded region?
   A. $X \cap Y \cap Z'$
   B. $X \cap Y \cup Z'$
   C. $X \cap Y \cap Z$
   D. $X \cup Y \cup Z$
6. Which of the following Venn diagrams illustrates $M \cap N = \emptyset$?

   a) ![Venn Diagram A]
   b) ![Venn Diagram B]
   c) ![Venn Diagram C]
   d) ![Venn Diagram D]

   C

---

Use the following information to answer the four questions

Students in a particular high school were surveyed to determine the subjects in which they were currently enrolled. The table below represents the data that was collected.

<table>
<thead>
<tr>
<th>Courses Enrolled In</th>
<th>Number of Students</th>
</tr>
</thead>
<tbody>
<tr>
<td>Math only</td>
<td>28</td>
</tr>
<tr>
<td>Art only</td>
<td>33</td>
</tr>
<tr>
<td>Math and Art</td>
<td>17</td>
</tr>
<tr>
<td>Neither course</td>
<td>20</td>
</tr>
</tbody>
</table>

---

7. The number of students in the universal set is
   A. 61
   B. 64
   C. 78
   D. 98

   D

8. The number of students taking Math or Art is
   A. 17
   B. 61
   C. 78
   D. 98

   C

**Numerical Response 1** The number of students taking Art is ________.

**Numerical Response 2** The number of students not taking Math is ________.
Use the following information to answer the next three questions

A group of 100 students was surveyed about movies that they have seen, as shown below.

- 2 people saw all three movies
- 12 people saw “Metal Man” and “The Princely Groom”
- 53 people saw “Metal Man”
- 10 people saw “Metal Man” and “Quick and Angry 8”
- 18 people saw “The Princely Groom” only
- 23 people saw “The Princely Groom” and “Quick and Angry 8”
- 6 people did not see any of the movies

Jason started to organize the results in the Venn diagram shown below.

9. The number of people who saw “The Princely Groom” is
   A. 18
   B. 20
   C. 51
   D. 53

10. The number of people who saw “Metal Man” or “Quick and Angry 8” is
    A. 10
    B. 43
    C. 76
    D. 98

Numerical Response 3 The number of people who saw “Metal Man” and “The Princely Groom” but not “Quick and Angry 8” is ____10____.
COUNTING METHODS

Specific Outcome 4
Solve problems that involve the fundamental counting principle (FCP)

EXAMPLE 1
A sub shop offers the following choices
- 3 types of buns
- 5 types of cold cuts
- 3 types of cheese
- 12 different toppings
- 3 different sauces
If Andrew wants a sub with one item from each category, how many different subs can he choose from?

\[
3 \times 5 \times 3 \times 12 \times 3 = 1620
\]

EXAMPLE 2
A student is writing a 10-question multiple-choice test. Each question has 4 choices: A to D. How many different sets of answers can the student give?

\[
4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4 = 4^{10} = 1,048,576
\]

Specific Outcome 5
Solve problems that involve permutations.

EXAMPLE 3
Evaluate

a) \[
\frac{9!}{6!} = \frac{9 \times 8 \times 7 \times 6!}{6!} = 84
\]
b) \[
\frac{10 \times 9!}{5 \times 8!} = \frac{10 \times 9 \times 8!}{3 \times 8!} = 18
\]
EXAMPLE 4
\[
\frac{(n + 2)!}{n!} = 20
\]
Solve
\[
\frac{(n+2)(n+1)n!}{n!} = 20
\]
\[
(n+2)(n+1)n = 20
\]
\[
(n+2)(n+1)n = 20
\]
\[
(n+6)(n-3) = 0
\]
\[
(n = -6, 3)
\]
\[
(n = 3)
\]
\[
\theta \cdot \frac{-3}{\theta - 3} = -18
\]
\[
\theta - 3 = 3
\]
\[
\theta = 6
\]

EXAMPLE 5
Jordan has 20 CDs in her car. Her CD player holds 6 CDs. How many different ways can she load her CD player?
Order matters
\[
20 19 18 17 16 15 \quad \text{or} \quad 20 \text{P}_6
\]
\[
2 + 907200
\]

EXAMPLE 6
Consider the 11 letters in the word MATHEMATICS. How many different arrangements are possible?
\[
\frac{11!}{2!2!} = \frac{11!}{2!2!} = 4989600 \text{ ways}
\]

EXAMPLE 7
A piano teacher and her students are having a group photo. There are three boys and five girls. The photographer wants the boys to sit together and the girls to sit together for one of the poses. How many ways can the students and teacher sit in a row of 9 chairs for this pose?
\[
3! \times \frac{5!}{1!} = 3! \times \frac{5!}{1!} = 4320
\]
EXAMPLE 8
How many different routes are there from A to B, if you can only travel north or west?

Specific Outcome 6
Solve problems that involve combinations.

EXAMPLE 9
How many ways can 6 players be chosen to start a volleyball game from a team of 10 players?

order doesn’t matter

\[ \binom{10}{6} = 210 \]

EXAMPLE 10
The town council is forming a committee, and 9 men and 10 women have volunteered. How many different ways can a committee of 4 people be chosen in each situation below?

a) There are no conditions.

\[ 9 C_4 \times 10 C_4 = 3876 \]

b) There must be an equal number of men and women on the committee.

\[ 9 C_2 \times 10 C_2 = 360 \times 45 = 1620 \]
PRACTICE TEST – COUNTING METHODS

1) A hockey arena has 8 gates. In how many ways can you enter the arena and leave the arena by a different gate?
   A. $8 \times 7$
   B. $8 + 8$
   C. $8^2$
   D. $8!$

2) Susan is playing a game of Scrabble. She picks the following 7 tiles from the bag. Use the following information to answer the next question.

   A. A
   B. R
   C. A
   D. B
   E. R

3) In Holland, license plates start with any two digits followed by any three letters except A, E, I, O, U, C and Q, followed by any single digit. If no letters are repeated, how many different license plates are possible with this configuration?
   A. $10 \times 10 \times 19 \times 18 \times 17 \times 10$
   B. $9 \times 9 \times 9 \times 19 \times 18 \times 17 \times 9$
   C. $10 \times 9 \times 19 \times 18 \times 17 \times 8$
   D. $10 \times 9 \times 19 \times 19 \times 19 \times 8$

4) Simplify: $\frac{(x-2)!}{(x-5)!5!}$
   A. $(x - 3)(x - 4)$
   B. $\frac{(x-3)(x-4)}{120}$
   C. $\frac{1}{120(x-5)(x-4)(x-3)}$
   D. $\frac{(x-2)(x-3)(x-4)}{120}$
5) In which step is Karla’s first mistake, if any? 
A. Step I  
B. Step II  
C. Step III  
D. There is no mistake.

Use the following information to answer the next question

A restaurant chef is planning a menu to show how 12 different dishes are possible by combining one vegetable, one starch, and a protein.

Numerical Response 1

If 2 different vegetables and 3 different proteins are used, then the minimum number of different starches required to show the 12 different dishes is ____________

Use the following information to answer the next question

A committee of 3 girls and 2 boys is to be chosen from a group of 9 girls and 7 boys. The total number of different committees that can be formed can be expressed in the form

\[ wC_x \times yC_z \]

where \( wC_x \) represents the number of possible choices of girls for the committee and \( yC_z \) represents the number of possible choices of boys for the committee.

Numerical Response 2

The values of \( w, x, y, \) and \( z \) are ______________, ____, ____, and ____, respectively.
Use the following information to answer the next question

Oscar is trying to determine the value of \( _5C_3 \) by listing the combinations of EFGHI. He chooses 3 letters at a time and creates the list shown below.

<table>
<thead>
<tr>
<th>EFG</th>
<th>FEG</th>
<th>GHI</th>
<th>IEG</th>
</tr>
</thead>
<tbody>
<tr>
<td>EFH</td>
<td>FEI</td>
<td>GEH</td>
<td>IGF</td>
</tr>
<tr>
<td>EFI</td>
<td>FGH</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

6) What is Oscar's mistake, if any?
   A. Oscar’s list represents permutations instead of combinations.
   B. Oscar’s list is incomplete and he has repeated one of the combinations.
   C. Oscar’s list is incomplete and he has repeated two of the combinations.
   D. There is no mistake. Oscar’s list shows all the combinations for \( _5C_3 \).

Use the following information to answer the next question

A student is classifying the following contexts that require the use of either permutations or combinations.

- **Context A** Dialing a 10-digit telephone number with distinct digits order matters
- **Context B** Choosing 5 people for a committee order doesn't matter
- **Context C** Selecting 4 fruits to put in a salad order doesn't matter
- **Context D** Opening a lock with a 3-number combination order matters

There is no mistake. Oscar’s list shows all the combinations for \( _5C_3 \).

Numerical Response 3

For each context, use a 1 to indicate if it should be solved using a permutation and use a 2 to indicate if it should be solved using a combination.

- Context A would be solved using a \( _A \) \( _B \) \( _C \) \( _D \)
- Context B would be solved using a \( _A \) \( _B \) \( _C \) \( _D \)
- Context C would be solved using a \( _A \) \( _B \) \( _C \) \( _D \)
- Context D would be solved using a \( _A \) \( _B \) \( _C \) \( _D \)

(Record in the first column)
(Record in the second column)
(Record in the third column)
(Record in the fourth column)
Use the following information to answer the next question

The Vimy Ridge Academy cafeteria offers milkshakes in 20 different flavors. The manager has decided to offer customers the choice of blending two different flavors together to create more flavor options.

\[ \binom{20}{2} = 190 \]

7) The total number of different possible two-flavor blended milkshakes is

A. 40
B. 190
C. 380
D. 400

Use the following information to answer the next question

Ethan is trying to get to his house by travelling only west and south along the streets shown in the diagram below.

Numerical Response 4

The number of different routes that Ethan can take is \[ \boxed{12} \].
Use the following information to answer the next question

Kiki is arranging 6 colored blocks in a row. There are 3 identical green blocks, 1 red block, 1 blue block, and 1 yellow block.

8) An expression that Kiki could use to calculate the total number of different possible arrangements of the 6 blocks is
   A. $6!$
   B. $3!$
   C. $\frac{6!}{3!}$
   D. $\frac{6!}{3!}$

   $\frac{6!}{3!} = \frac{720}{6} = 120$

Use the following information to answer the next question

The back of a credit card shows a 3-digit card verification code (CVC). The digits can be any number from 0 through 9, and digits may be repeated.

**Numerical Response 5**

If the number of digits in the CVC was changed from 3 digits to 4 digits, then the number of possible CVCs would increase by 9,000.

$10 \times 10 \times 10 \times 10 = 10,000$

Increase by 9,000
Specific Outcome 1
Interpret and assess the validity of odds and probability statements.

Example 1

Use the following information to answer the next question.

The Probability and Odds of falling down while snowboarding in three students.

<table>
<thead>
<tr>
<th>Student</th>
<th>Description</th>
<th>Odds</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bennett</td>
<td>The odds in favor of falling down while snowboarding are 14:11</td>
<td>( \frac{14}{25} = 0.56 )</td>
</tr>
<tr>
<td>Carter</td>
<td>The odds against of falling down while snowboarding are 8:17</td>
<td>( \frac{17}{25} = 0.68 )</td>
</tr>
<tr>
<td>David</td>
<td>The probability of falling down while snowboarding is 0.61</td>
<td></td>
</tr>
</tbody>
</table>

If the snowboarders listed above are arranged from the lowest probability to the highest probability of a snowboarder falling down, then the order will be

Bennett, David, then Carter.

Example 2

Jamaal, Ethan, and Alberto are competing with seven other boys to be on their school’s cross-country team. All the boys have an equal chance of winning the trial race. Determine the probability that Jamaal, Ethan, and Alberto will place first, second, and third, in any order.

\[
P = \frac{\text{total}}{\text{favorable}} = \frac{6}{10 P_3} = \frac{6}{\frac{6!}{3!1!}} = \frac{6}{20} = 0.30
\]

or \( 0.30 \) or \( 30\% \)
Example 3

Beau hosts a morning radio show in Saskatoon. To advertise his show, he is holding a contest at a local mall. He spells out SASKATCHewnA with letter tiles. Then he turns the tiles face down and mixes them up. He asks Sally to arrange the tiles in a row and turn them face up. If the row of tiles spells SASKATCHewan, Sally will win a new car. Determine the probability that Sally will win the car.

\[
P(\text{win}) = \frac{1}{39916800}
\]

Specific Outcome 2
Solve problems that involve the probability of mutually exclusive and non-mutually exclusive events.

Example 4

Use the following information to answer the next question.

Some possible events for rolling a regular six-sided die are listed below
1. An even number: 2, 4, 6
2. A number less than 3: 1, 2
3. A number that is greater than or equal to 2: 2, 4, 6
4. A number that is a multiple of 3: 3, 6

From the list above, the two events that are mutually exclusive are numbered 2 and 4.

Example 5

A particular traffic light at the outskirts of a town is red for 30 seconds, green for 25 seconds, and yellow for 5 seconds in every minute. When a vehicle approaches the traffic light, the probability that the light will be red or yellow is

\[
P(R \text{ or } Y) = P(R) + P(Y) = \frac{30}{60} + \frac{5}{60} = \frac{35}{60} = \frac{7}{12}
\]
Example 6
Pearl is about to draw a card at random from a standard deck of 52 playing cards. If she draws a face card or a spade, she will win a point. Determine the probability of drawing a face card or a spade.

\[ P(\text{Face or Spade}) = P(\text{Face}) + P(\text{Spade}) - P(\text{Face and Spade}) \]

\[ = \frac{12}{52} + \frac{13}{52} - \frac{3}{52} \]

\[ = \frac{22}{52} = \frac{11}{26} \]

Example 7
Use the following information to answer the next question.
The probability of Brenda getting hit in a baseball game is 0.345. The probability of Brenda or Deborah getting a hit during the game is 0.617. The probability of both Brenda and Deborah getting hits during the game is 0.224.

Determine the probability of Deborah getting a hit in the game.

\[ P(B) = 0.345 \]

\[ P(B \cup D) = 0.617 \]

\[ P(B \cap D) = 0.224 \]

\[ P(D) = P(B \cup D) - P(B) = 0.617 - (0.345 + 0.224) = 0.048 \]

Specific Outcome 3
Solve problems that involve the probability of two events.

Example 8
For each situation, classify the events as either independent or dependent.

a) A four-colour spinner is spun, and a die is rolled. The first event is spinning red, and the second event is rolling a 2.

\[ \text{Independent} \]

b) Two cards are drawn, without being replaced, from a standard deck of 52 playing cards. The first event is drawing a king, and the second event is drawing an ace.

\[ \text{Dependent} \]
Example 9

Use the following information to answer the next question.

A child has a box containing 8 toy cars and 6 toy trucks. He reaches in and randomly selects 2 toys, one after the other, without replacement.

Determine the probability that the two toys selected are both toy cars.

\[
P(C \cap C) = P(C) \times P(C|C)
\]

\[
= \frac{8}{14} \times \frac{7}{13}
\]

\[
= \frac{4}{13}
\]

Example 10

Use the following information to answer the next question.

In a particular class, the probability that a student has a home video game system is 0.62. In the same class, the probability that a student will have a home video game system and a TV in their bedroom is 0.46.

Assuming that having a home video game system and having a TV in their bedroom are independent, determine the probability, to the nearest hundredth, that a student in the class has a TV in their bedroom.

\[
P(V \cap TV) = P(V) \cdot P(TV)
\]

\[
0.46 = 0.62 \cdot P(TV)
\]

\[
\frac{0.46}{0.62} = P(TV)
\]

\[
\frac{23}{31} = P(TV)
\]

\[
0.74 = P(TV)
\]
Example 11

Each day, Melissa’s math teacher gives the class a warm-up question. It is a true and false question 30% of the time and a multiple-choice question 70% of the time. Melissa gets 60% of the true-false questions correct, and 80% of the multiple-choice questions correct. What is the probability that Melissa answers today’s question correctly?

\[
\begin{align*}
\text{Question} & \quad \text{Correct} \\
0.3 & \quad P(\text{true}) \quad P(\text{correct}) \rightarrow 0.18 \\
0.4 & \quad P(\text{false}) \quad P(\text{wrong}) \\
0.7 & \quad P(\text{mc}) \quad P(\text{correct}) \rightarrow 0.56 \\
& \quad P(\text{wrong}) \\
0.18 + 0.56 &= 0.74
\end{align*}
\]
PRACTICE TEST - PROBABILITY

Use the following information to answer the next question.

A survey of Alberta students was used to estimate the percentage of students who participate in school and non-school athletics, as well as the percentage who do not participate in any athletics. The results of the survey are shown in the Venn diagram below.

Participation in school and non-school athletics.

1. The odds in favor of randomly selecting an Alberta student who participates in school athletics are:
   A. 15:64
   B. 15:85
   C. 53:26
   D. 53:47

Use the following information to answer the next question.

<table>
<thead>
<tr>
<th></th>
<th>Odds For</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>1 : 2</td>
<td>$\frac{1}{3}$</td>
</tr>
<tr>
<td>II</td>
<td>3 : 2</td>
<td>$\frac{3}{5}$</td>
</tr>
<tr>
<td>III</td>
<td>4 : 6</td>
<td>$\frac{2}{5}$</td>
</tr>
</tbody>
</table>

2. Which of the following “odds for” and “probability” statements are equivalent?
   A. I only
   B. II only
   C. I and II only
   D. II and III only
Use the following information to answer the next question.

There are 10 black, 12 red, 14 yellow, 9 green, 8 orange, and 11 purple candies in a particular package of candy.

\[
P(C\cup R\cup B) = P(R) + P(B) = \frac{12}{64} + \frac{10}{64} = \frac{22}{64}.
\]

Numerical Response 1

If one candy is randomly selected, then the probability, to the nearest hundredth, that it is a red candy or a black candy is \(0.34\).

Use the following information to answer the next question.

Marco represents an entire sample space (S) with the Venn diagram below. Each X represents a possible outcome for events P and Q.

3. Which statement below is true?
   A. Q is the complement of P.
   B. The probability of P and Q is 2/19
   C. P and Q are not mutually exclusive.
   D. Event Q is twice as likely to occur as event P.

C
4. What is the probability that her birthday is on Valentine’s Day (February 14) or on a weekend (Saturday or Sunday)?

A. \( \frac{4}{28} \)  
B. \( \frac{8}{28} \)  
C. \( \frac{9}{28} \)  
D. \( \frac{8}{35} \)

5. A particular traffic light at the outskirts of a town is red for 30 s, green for 25 s, and yellow for 5 s in every minute. When a vehicle approaches the traffic light, the probability that the light will be red or yellow is

A. \( \frac{7}{12} \)  
B. \( \frac{1}{2} \)  
C. \( \frac{1}{12} \)  
D. \( \frac{1}{24} \)

6. In a biological study on genetically modified mice, 45% have blue eyes, 30% have a short tail, and 20% have both blue eyes and a short tail. What is the probability that a randomly selected mouse from this study has neither blue eyes nor a short tail?

A. 5%  
B. 25%  
C. 45%  
D. 75%
7. A soccer team has practice jerseys in three different colours. The team bag contains 4 yellow, 6 white and 5 orange jerseys. Beck randomly gives Jessica and Victoria each a jersey. Which expression correctly represents the probability that both jerseys are the same colour?

A. \( \left( \frac{2}{4} \right) \left( \frac{2}{6} \right) \left( \frac{2}{5} \right) \)

B. \( \left( \frac{2}{4} \right) + \left( \frac{2}{6} \right) + \left( \frac{2}{5} \right) \)

C. \( \left( \frac{4}{15} \right) + \left( \frac{6}{15} \right) + \left( \frac{5}{15} \right) \)

D. \( \left( \frac{4}{15} \right) \left( \frac{4}{15} \right) + \left( \frac{6}{15} \right) \left( \frac{6}{15} \right) + \left( \frac{5}{15} \right) \left( \frac{5}{15} \right) \)

8. The probability that Naomi will make a free throw in basketball is 0.53. The probability that Jacob will make a free throw is 0.33. Assume independence. What is the probability that at least one of them will make a free throw on their next shot?

A. 0.31
B. 0.69
C. 0.83
D. 0.86

9. Hayden is about to draw two cards from a pile of cards (with replacement). The pile contains only jacks, queens and kings from a standard deck. What is the probability that Hayden draws two cards that are both kings or both red cards?

A. \( \left( \frac{1}{3} \times \frac{1}{3} \right) + \left( \frac{1}{2} \times \frac{1}{2} \right) \)

B. \( \left( \frac{1}{3} \times \frac{1}{3} \right) + \left( \frac{1}{2} \times \frac{1}{2} \right) - \left( \frac{1}{6} \times \frac{1}{6} \right) \)

C. \( \left( \frac{1}{3} \times \frac{1}{3} \right) + \left( \frac{1}{2} \times \frac{1}{2} \right) - \left( \frac{1}{6} \times \frac{1}{2} \right) \)

D. \( \left( \frac{1}{3} \times \frac{1}{3} \right) + \left( \frac{1}{2} \times \frac{1}{2} \right) - \left( \frac{1}{6} \times \frac{1}{6} \right) \)
RATIONAL EXPRESSIONS

Specific Outcome 1
Determine equivalent forms of rational expressions (limited to numerators and denominators that are monomials and binomials).

Example 1

\[
\frac{8a^2}{2a-6} = \frac{4a^2}{2(a-3)} = \frac{4a^2}{a-3}, \quad a \neq 3
\]

Example 2

\[
\frac{36y^2 - 1}{12y^2 - 2y} = \frac{(6y+1)(6y-1)}{2y(6y-1)} = \frac{6y+1}{2y}, \quad y \neq 0, \frac{1}{6}
\]

Specific Outcome 2
Perform operations on rational expressions (limited to numerators and denominators that are monomials and binomials).

Example 3

\[
\frac{m^2 + 2m}{m^2 - 4} \cdot \frac{12m - 24}{16m} = \frac{m(m+2)}{(m-2)(m+2)} \cdot \frac{12(m-2)}{16m} = \frac{12}{16} = \frac{3}{4}, \quad m \neq \pm 2, 0
\]
Example 4

Simplify $\frac{6x^2}{15x^3} \div \frac{4x(x-1)}{20x^2}$

$\frac{6x^2}{15x^3} \rightarrow \frac{20x^2}{4x(x-1)} = \frac{2}{x-1}, x \neq 0, 1$

Example 5

Simplify $\frac{5x+1}{x-1} - \frac{3x-1}{x-1}$

$= \frac{(5x+1) - (3x-1)}{x-1} = \frac{5x+1-3x+1}{x-1}$

$= \frac{2x+2}{x-1} = \frac{2(x+1)}{x-1}, x \neq 1$

Example 6

Simplify $\frac{3}{x+1} - \frac{2}{3x}$

$\frac{3}{x+1} \cdot \frac{3x}{3x} - \frac{2}{3x} \cdot \frac{(x+1)}{x+1} = \frac{9x-2(x+1)}{3x(x+1)} = \frac{9x-2x-2}{3x(x+1)}$

$= \frac{7x-2}{3x(x+1)}, x \neq 0, -1$
Specific Outcome 3
Solve problems that involve rational equations (limited to numerators and denominators that are monomials and binomials).

Example 7

Solve \( \frac{4}{x} + \frac{6x}{x+1} = 6 \)

\[
\frac{4(x+1)}{x(x+1)} + \frac{6x(x)}{(x+1)(x)} = \frac{6(x)(x+1)}{(x)(x+1)}
\]

\[
4(x+1) + 6x(x) = 6x(x+1)
\]

\[
4x + 4 + 6x^2 = 6x^2 + 6x
\]

\[
4x + 4 = 6x
\]

\[
-x = \frac{4}{4} - \frac{4}{4}
\]

\[
4 = +2x
\]

\[
+2 = \frac{4}{2}
\]

\[
+2 = x
\]
1) An equivalent form of the expression \( \frac{12x^2 - 6x^3}{4x^3 - 2x^4}, x \neq 0,2 \) is

A. \( 3x \)
B. \( -3x \)

2) When the rational expression \( \frac{2x + 4}{x^2 - 4} \) is simplified, the equivalent expression is

A. \( \frac{2}{x-2}, x \neq -2,2 \)
B. \( \frac{2}{x+2}, x \neq -2 \)

Use the following information to answer the next question.

A student is working on simplifying the expression \( \frac{2x^2 - 18}{12 - 6x} \div \frac{2x + 10}{x^2 - 4} \). Six statements are made regarding the non-permissible values of this expression.

- Statement 1: \( x = 3 \)
- Statement 2: \( x = -3 \)
- Statement 3: \( x = 2 \)
- Statement 4: \( x = -2 \)
- Statement 5: \( x = 5 \)
- Statement 6: \( x = -5 \)

The three statements above that represent the non-permissible values of the expression are statements 3, 4, and 6.
3) When \( \frac{x}{3x+12} + \frac{x-1}{6x+24} \), \( x \neq -4 \), is simplified, the numerator is \( 3x - 1 \) and the denominator is

(A) \( 6(x+4) \)
(B) \( 6(x+4)^2 \)
(C) \( 18(x+4) \)
(D) \( 18(x+4)^2 \)

\[ \frac{x}{3(x+4)} + \frac{x-1}{6(x+4)} \]
\[ \frac{2x}{6(x+4)} + \frac{x-1}{6(x+4)} \]
\[ \frac{3x-1}{6(x+4)} \]

Use the following information to answer the next question.

The simplified product of \( \frac{2n^4 p}{3m} \cdot \frac{6m^6}{3n^2 p^2} \), \( m \neq 0, n \neq 0, p \neq 0 \), can be represented by

\[ \frac{An^b m^c}{3p} \]
\[ \frac{12m^n p}{9mn^3 p^2} \]
\[ \frac{4mn^2}{3p} \]

where A, B, and C represent single-digit numbers.

**Numerical Response 2**

In the simplified product \( \frac{An^b m^c}{3p} \), the value of

A is \[ 4 \]
B is \[ 2 \]
C is \[ 5 \]

4) The solution for \( x \) in the equation \( \frac{-2}{3} - \frac{4}{x} = 6, x \neq 0 \) is

A. \(-15\)
B. \(-9\)
C. \(3\)
D. \(\frac{3}{5}\)

\[ \frac{-2}{3} \cdot x - \frac{4 \cdot 3}{3} = \frac{6 \cdot 3x}{3x} \]
\[ \frac{-2x - 12}{3x} = \frac{18x}{3x} \]

\[ -2x - 12 = 18x \]
\[ + 2x \]
\[ \frac{-12}{20} \]
\[ \frac{20x}{20} \]

\[ -\frac{3}{5} = x \]
Use the following information to answer the next question.

A contractor and a gravel-truck driver arranged to meet at a sandpit 9 km away. In travelling to the pit, the gravel truck's average speed was 25 km/h slower than that of the contractor's car. The table below summarizes this information.

<table>
<thead>
<tr>
<th>Distance Travelled (km)</th>
<th>Average Speed (km/h)</th>
<th>Trip Time (h)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Car</td>
<td>9</td>
<td>( \frac{9}{x} )</td>
</tr>
<tr>
<td>Truck</td>
<td>9</td>
<td>( \frac{9}{x-25} )</td>
</tr>
</tbody>
</table>

The gravel truck took 4 minutes longer than the car to reach the gravel sandpit, as modelled by the equation below.

\[
\frac{9}{x-25} - \frac{9}{x} = \frac{4}{60}, \quad \frac{60 \cdot 9x - 60 \cdot 9(x-25)}{60 \cdot x(x-25)} = \frac{4 \cdot x(x-25)}{60 \cdot x(x-25)}
\]

**Numerical Response**

\[
\therefore \quad 540x - 540(x-25) = 4x(x-25)
\]

\[
540x - 540x + 13500 = 4x^2 - 100x
\]

\[
0 = 4x^2 - 100x - 13500
\]

\[
0 = 4(x^2 - 25x - 3375)
\]

To the nearest kilometre per hour, the average speed of the car, \( x \), was \( \frac{72}{x-25} \) km/h.

**Use the following information to answer the next question.**

The rational equation \( \frac{2 + x}{x + 2} = \frac{3}{x} \) is solved for \( x \) using the work shown below.

\[
x(2 + x) = 3(x + 2)
\]

\[
x^2 + 2x = 3x + 6
\]

\[
x^2 - x - 6 = 0
\]

\[(x - 3)(x + 2) = 0
\]

\[x = 3 \text{ or } x = -2\]

5) Which statement correctly completes the solution

A. Both \( x = 3 \) and \( x = -2 \) are valid solutions
B. Both \( x = 3 \) and \( x = -2 \) are extraneous solutions
C. The solution \( x = 3 \) is a valid solution, but \( x = -2 \) is an extraneous solution
D. The solution \( x = -2 \) is a valid solution, but \( x = 3 \) is an extraneous solution

\[
\frac{2 + 3}{3 + 2} = \frac{3}{3} \quad \frac{5}{5} = \frac{3}{3}
\]

\[
\frac{2 + (-2)}{(-2) + 2} = \frac{-2}{-2} \quad \frac{0}{0} = \frac{3}{3}
\]

-2 not valid.
Exponential & Logarithmic Functions

Specific Outcome 4
Demonstrate an understanding of logarithms and the laws of logarithms.

Example 1
Convert to exponential and solve

a) \( \log_8 64 = x \)
\[
8^x = 64
\]
\[
8^x = 8^2
\]
\[
x = 2
\]

b) \( \log_{\frac{1}{4}} 16 = x \)
\[
\left(\frac{1}{4}\right)^x = 16
\]
\[
\left(4^{-1}\right)^x = 4^2
\]
\[
4^{-x} = 4^2
\]
\[
-x = 2
\]

Example 2
Write each expression as a single logarithm, and then evaluate.

a) \( \log_6 108 - \log_6 3 \)
\[
\log_6 \left(\frac{108}{3}\right)
\]
\[
\log_6 36
\]
\[
2
\]

b) \( \log 4 + 2 \log 5 \)
\[
\log 4 + \log 5^2
\]
\[
\log 100 = 2
\]

Example 3
Evaluate \( \log_8 146 \) using calculator

\[
\frac{\log 146}{\log 8}
\]
Example 4

The pH of a solution, \( p(x) \), can be determined using the function

\[
p(x) = -\log x
\]

where \( x \) represents the hydrogen ion concentration of the solution in moles/litre.

a) Determine the pH of a solution that has a hydrogen ion concentration of 0.005 mol/L.

\[
p(0.005) = -\log (0.005)
\]

\[
\text{pH} = 2.3
\]

b) Black coffee has a pH of 5, and bleach has a pH of 13. In terms of their hydrogen ion concentrations, how much more acidic is black coffee than bleach?

\[
\frac{10^5}{10^{13}} = 10^{5-13} = 10^{-8}
\]

coffee is \( 10^8 \) times more acidic than bleach.

Example 5

Use the following information to answer the next question.

The 2010 earthquake in Haiti measured 7.0 on the Richter scale. In 2004, an earthquake off Indonesia measuring 9.1 on the same scale cause a tsunami. In order to compare the relative intensities, \( I \), of these two earthquakes, the following formula may be used.

\[
I = \frac{10^m}{10^n}
\]

Where \( m \) and \( n \) are the Richter scale values of each earthquake.

Compare to the earthquake in Haiti, the earthquake off Indonesia was ______ times more stronger.

\[
\frac{10^{9.1}}{10^7} = 10^{9.1-7} = 10^{2.1}
\]

\[
125,890
\]
Specific Outcome 5
Solve problems that involve exponential equations.

Example 6

\[ 7^x = \left( \frac{1}{49} \right)^{x+3} \]

Solve

\[ 7 = \left( 7^{-2} \right)^{x+3} \]
\[ 7 = 7^{-2x-6} \]
\[ 2x + 6 = \]  
\[ x = -3 \]

Example 7

Solve \( 3 = 8^{2x-1} \)

\[ 2x - 1 = \log_2 8^{3+1} \]
\[ 2x = \log_2 \frac{8^{3+1}}{2} \]
\[ x = \frac{105}{2} \]
\[ x = 52.5 \]

Example 8

Ahmed invested $1000 for four years into an account that pays 3%/a, compounded annually. Determine a function expression that models the value, \( V \), of Ahmed’s investment.

\[ V = 1000 \left( 1 + \frac{0.03}{1} \right)^{4.1} \]

\[ V = 125.50 \]
Example 9

Use the following information to answer the next question.

From 1997 to 2008, the population of a particular town grew at an average rate of 5.4%/a. During this 11 year period, its population, $P$, can be modelled by the exponential function

$$ P = x(1 + r)^n $$

Where $x$ is the population in 1997, $r$ is the average annual growth rate, and $n$ is the number of years since 1997. In 2008, the population was 14 030.

According to the model, the population of this town in 1997 was approximately _________.

\[
14030 = x \left( 1 + 0.054 \right)^{11} \\
14030 = x \left( 1.054 \right)^{11} \\
(1.054)^{11} = x \\
7867 = x
\]

Specific Outcome 6

Represent data, using exponential and logarithmic functions, to solve problems.

Example 10

Use each equation to predict the x-intercept, the number of y-intercepts, the end behaviour, the domain, and the range of the function. Verify your predictions with graphing technology.

a) \( y = 125(0.78)^x \)

\( x: [-10, 10] \)
\( y: [0, 2000, 1] \)

No x int
\( y \) int = 125 (True 0)

D: x > 0 R: y > 0
II \( \rightarrow \) I

b) \( y = 25 \log x \)

\( x: [-10, 10, 1] \)
\( y: [-10, 10, 1] \)

No x int
\( y \) int = 0 (True 0)

D: x > 0 R: y > 0
I \( \rightarrow \) II

\( 39 \) Page
Example 11

A naturalist group has been tracking the deer population near the town of Hudson Bay, Saskatchewan, since 2005.

<table>
<thead>
<tr>
<th>Years After 2005</th>
<th>Deer Population</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>218</td>
</tr>
<tr>
<td>2</td>
<td>237</td>
</tr>
<tr>
<td>3</td>
<td>260</td>
</tr>
<tr>
<td>4</td>
<td>282</td>
</tr>
<tr>
<td>5</td>
<td>307</td>
</tr>
<tr>
<td>6</td>
<td>336</td>
</tr>
<tr>
<td>7</td>
<td>360</td>
</tr>
</tbody>
</table>

a) Determine the equation of the exponential regression function that models the data.

\[ y = 200.798 (1.088)^x \]

b) Assuming the same growth rate, predict the deer population 10 years after 2005.

2nd trace \( R^1 \) \( \text{value} \)

467.97 \( \Rightarrow \) \( 468 \) deer

\( X = 10 \)

\( x = 10 \) years after 2005

(c) In which year would you expect the population to have doubled from its 2005 level?

2nd trace \( R^1 \) \( \text{value} \) 0 enter \( \Rightarrow \) 200 deer

(a' value is initial amount)

\[ y_2 = 400 \]

2nd Calc \( S: \text{Intersect} \)

\( x = 8.2 \) years after 2005

\( \therefore \) in the year 2013
Example 12

The euphotic zone is the upper 200m layer of the oceans. Very little sunlight penetrates deeper than 200, so most plants live in the euphotic zone. As a result, 70% of all photosynthesis on Earth occurs in the euphotic zone of the oceans. The following table gives light penetration data for the location in the Pacific Ocean.

<table>
<thead>
<tr>
<th>Depth (m)</th>
<th>Penetration of Sunlight (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>100</td>
</tr>
<tr>
<td>20</td>
<td>54.37</td>
</tr>
<tr>
<td>40</td>
<td>29.57</td>
</tr>
<tr>
<td>60</td>
<td>16.08</td>
</tr>
<tr>
<td>80</td>
<td>8.74</td>
</tr>
<tr>
<td>100</td>
<td>4.76</td>
</tr>
</tbody>
</table>

a) Determine the equation of the logarithmic regression function that best models the data. Round your values to the nearest hundredth.

\[ y = 151.21 - 33.84 \ln x \]

b) Using your equation, determine the depth with a sunlight penetration of 35%. Express your answer to the nearest tenth of a meter.

\[ x = 35 \%
\]

\[ y = 34.46 \text{ m} \]

c) Using your equation, determine the amount of sunlight that penetrates to a depth of 200m at this location. Express your answer to the nearest hundredth of a percent. Describe your process.

\[ y_2 = 2.00 \]

2nd Trace 5:Intersect

0.23%
1) Which logarithmic equation correctly represents the exponential equation $10^7 = x$?
A. $x = \log 7$
B. $x = \log 10$
C. $7 = \log x$
D. $10 = \log x$

2) Which of the following logarithmic expressions is equivalent to $7 = 6^{(2x+3)}$?
A. $\log_{6}(2x + 3) = 7$
B. $\log_{6}7 = 2x + 3$
C. $\log_{7}(2x + 3) = 6$
D. $\log_{7}6 = 2x + 3$

3) If $5\log_{4}(x) = 5$, then $x$ is:
A. 5
B. 4
C. 3
D. 2

4) When $\log_{b}(3x) + \log_{b}(x^5) - \log_{b}(6x^2)$ is expressed as a single logarithm, the result is
A. $\log_{b}(18x^9)$
B. $\log_{b}(18x^{12})$
C. $\log_{b}\left(\frac{x^4}{2}\right)$
D. $\log_{b}\left(\frac{1}{2x^7}\right)$

**Numerical Response**

A partial graph of $y = \log_{a}x$ passes through the point $(8, 1.5)$. Correct to the nearest whole number, the value of $a$ is __4__

\[
1.5 = \log_{a}8 \quad \Rightarrow \quad \left(8^{\frac{3}{2}}\right) = (8) \quad \Rightarrow \quad a^{1.5} = 8 \quad \Rightarrow \quad \left(\frac{3}{2}\right) = (8) \quad \Rightarrow \quad a = 4
\]
5) Solve the following exponential equation: \(4^{x+2} = 1024\)
   A. \(z = 0\)
   B. \(z = 1\)
   C. \(z = 2\)
   D. \(z = 3\)

6) An expression that is equivalent to \(3 \log a - 3 \log b\) is:
   A. \(\log(a - b)^3\)
   B. \(3 \log(a - b)\)
   C. \(\frac{a^3}{b^3}\)
   D. \(\log\left(\frac{a}{b}\right)^3\)

7) Solve the following exponential equation: \(3^{4a} = \sqrt{243}\)
   A. \(a = \frac{3}{4}\)
   B. \(a = \frac{5}{8}\)
   C. \(a = \frac{9}{16}\)
   D. \(a = \frac{7}{8}\)

Use the following information to answer the next question:
Sam deposits $100 into a savings account that pays 2.4%/a, compounded monthly. A function that models the growth of the deposit is:

\[y = 100\left(1 + \frac{0.024}{12}\right)^x\]

where \(x\) = number of months and \(y\) = value of investment, in dollars.

**Numerical Response 2**

Determine, to the nearest month, how long it will take for the investment to be worth at least $150 at 2.4%/a, compounded monthly.

Use the following information to answer the next question:

The pH of a solution can be determined using the formula:

\[\text{pH} = -\log[C]\]

where \(C\) is the concentration of hydrogen ions in the solution. The pH of a particular solution is 6.6.

**Numerical Response 3**

To the nearest tenth, if the concentration of hydrogen ions in the solution is doubled, the new pH of the solution will be \(6.3\).

\[
\begin{align*}
6.6 &= -\log C \\
-6.6 &= \log C \\
10^{-6.6} &= C \\
2.51 \times 10^{-7} &= C \\
\end{align*}
\]

\[
\begin{align*}
\text{new } C &= 2(2.51 \times 10^{-7}) \\
\text{new } C &= 5.02 \times 10^{-7} \\
\text{pH} &= -\log(5.02 \times 10^{-7}) \\
\text{pH} &= 6.3
\end{align*}
\]
8) Match the equations with the graphs in the tables below.

<table>
<thead>
<tr>
<th>Equation I</th>
<th>$y = 3\left(\frac{1}{2}\right)^x$</th>
<th>Equation II</th>
<th>$y = \frac{1}{3}(2)^x$</th>
<th>Equation III</th>
<th>$y = \ln x$</th>
<th>Equation IV</th>
<th>$y = \ln(x) + 6$</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>Graph P</th>
<th>![Graph P Image]</th>
<th>Graph Q</th>
<th>![Graph Q Image]</th>
<th>Graph R</th>
<th>![Graph R Image]</th>
<th>Graph S</th>
<th>![Graph S Image]</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>Equation I</th>
<th>Equation II</th>
<th>Equation III</th>
<th>Equation IV</th>
</tr>
</thead>
<tbody>
<tr>
<td>Graph R</td>
<td>Graph P</td>
<td>Graph Q</td>
<td>Graph S</td>
</tr>
<tr>
<td>Graph P</td>
<td>Graph R</td>
<td>Graph Q</td>
<td>Graph S</td>
</tr>
<tr>
<td>Graph R</td>
<td>Graph P</td>
<td>Graph S</td>
<td>Graph Q</td>
</tr>
<tr>
<td>Graph P</td>
<td>Graph R</td>
<td>Graph S</td>
<td>Graph Q</td>
</tr>
</tbody>
</table>
Use the following information to answer the next question

A researcher discovered mould growing in a Petri dish in her laboratory. When first observed, the mould covered only 3% of the dish’s surface. Every 24 hours, the surface area of the mould doubles in size, as shown in the table below.

<table>
<thead>
<tr>
<th>Time (h)</th>
<th>Area covered (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>24</td>
<td>6</td>
</tr>
<tr>
<td>48</td>
<td>12</td>
</tr>
<tr>
<td>72</td>
<td>24</td>
</tr>
</tbody>
</table>

\[ y = 3(1.029)^t \]

**Numerical Response 4** Complete the table above and then write an exponential function to model the growth of the mould over time. Use your function to determine the approximate length of time, to the nearest hour, it will take for the Petri dish to be completely covered with mould.  

\[ t = 100 \]  
\[ y = 100 \]  
\[ \text{find intersection} \]  
\[ x = 121 \text{ hours} \]

Use the following information to answer the next question

When objects of different mass are compared without a scale, to be perceived the difference in mass must be large enough. For example, when held in a person’s hands, masses within 5 g of 100 g will seem to be the same. The 5 g difference is known as the Minimum Perceivable Difference.

For heavier objects, the Minimum Perceivable Difference increases. The Minimum Perceivable Difference for various masses is shown in the table below.

<table>
<thead>
<tr>
<th>Mass (g)</th>
<th>Minimum Perceivable Difference (g)</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>5</td>
</tr>
<tr>
<td>200</td>
<td>10</td>
</tr>
<tr>
<td>400</td>
<td>15</td>
</tr>
<tr>
<td>800</td>
<td>20</td>
</tr>
</tbody>
</table>

These data can be modelled by a logarithmic regression function of the form  
\[ y = a + b \ln(x) \]

where \( x \) is the mass of the object, in grams, and \( y \) is the Minimum Perceivable Difference in mass, in grams.

\[ y = -28.22 + 7.21 \ln x \]

**Numerical Response 5**

Based on the regression equation, determine the Minimum Perceivable Difference for a 2100 g object, to the nearest whole gram.

\[ y = 26.96 \approx 27 \text{ grams} \]
Polynomial Functions

Specific Outcome 7
Represent data, using polynomial functions (of degree ≤ 3), to solve problems.

EXAMPLE 1

Without graphing, determine the following characteristics of each function using its equation.

- Number of possible x-intercepts = degree
- Domain
- y-intercept

a) \( f(x) = 3x - 5 \) → Linear
  - 1 possible x-intercept
  - set \( y = 0 \):
    \[ 0 = 3x - 5 \]
    \[ \frac{5}{3} = x \]
  - \( x \in \mathbb{R} \)
  - y-intercept = -5

b) \( f(x) = -2x^2 - 4x + 8 \) → quadratic opening down
  - 2 possible x-intercepts
  - \( x \in \mathbb{R} \)
  - y-int = 8
  - range \( y \leq \max \), \( y \leq 10 \)
  - \( Q_3 \rightarrow Q_1 \)
  - 1 possible turning point

(c) \( f(x) = 2x^3 + 10x^2 - 2x - 10 \) → cubic opening up
  - 3 possible x-intercepts
  - \( x \in \mathbb{R} \)
  - y-int = -10
  - range \( y \in \mathbb{R} \)
  - \( Q_3 \rightarrow Q_1 \)
  - 2 possible turning points
EXAMPLE 2

Matt buys T-shirts for a company that prints art on T-shirts and then resells them. When buying the T-shirts, the price Matt must pay is related to the size of the order. Five of Matt’s past orders are listed in the table below.

<table>
<thead>
<tr>
<th>Number of Shirts</th>
<th>Cost per Shirt ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>500</td>
<td>3.25</td>
</tr>
<tr>
<td>700</td>
<td>1.95</td>
</tr>
<tr>
<td>200</td>
<td>5.20</td>
</tr>
<tr>
<td>460</td>
<td>3.51</td>
</tr>
<tr>
<td>740</td>
<td>1.69</td>
</tr>
</tbody>
</table>

a) Determine an equation for the linear regression function that models the data.

\[ y = -0.00065x + 6.5 \]

b) Use the linear regression function to achieve the price of $1.50 per shirt.

\[
\begin{align*}
1.5 &= -0.00065x + 6.5 \\
-5 &= -0.00065x \\
-0.00065 &= -0.00065x
\end{align*}
\]

EXAMPLE 3

Mr. Tran hit a golf ball from the top of a hill. The height of the ball above the green is given in the table below.

<table>
<thead>
<tr>
<th>Time (s)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Height (m)</td>
<td>52.5</td>
<td>73.2</td>
<td>74.6</td>
<td>55.8</td>
<td>16.1</td>
</tr>
</tbody>
</table>

a) Determine the equation of the quadratic regression function that models the data.

\[ y = -10.07x^2 + 51.41x + 11 \]

b) Use your equation to determine the height of the ball at 4.5 sec.

\[ y = -10.07(4.5)^2 + 51.41(4.5) + 11 \]

\[ y = 38.4 \text{ m} \]

C) When did the ball hit the ground?

\[ h(t) = -16t^2 + 38.4t + 11 \]

\[ 0 = -16t^2 + 38.4t + 11 \]

\[ t = 5.31 \text{ s} \]
EXAMPLE 4

The following table shows the number of females who entered a trade program in Canada in selected years after 1990.

<table>
<thead>
<tr>
<th>Years after 1990</th>
<th>Number of Females</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>8 245</td>
</tr>
<tr>
<td>5</td>
<td>11 425</td>
</tr>
<tr>
<td>7</td>
<td>13 305</td>
</tr>
<tr>
<td>9</td>
<td>15 675</td>
</tr>
<tr>
<td>13</td>
<td>24 280</td>
</tr>
<tr>
<td>15</td>
<td>28 755</td>
</tr>
<tr>
<td>17</td>
<td>38 070</td>
</tr>
</tbody>
</table>

Statistics Canada

a) Determine the equation for the cubic regression function that models the data.

\[ y = 7.06x^3 - 77.35x^2 + 1069.99x + 7208 \]

\( R^2 = 0.99 \)

b) Use your equation to estimate the year in which the number of females who entered a trade program was 20 000.

\[ y_2 = 20 000 \]

\( x = 11.48 \)

1990 + 11.48

in year 2001
1) Which of the following graphs would most likely represent the graph of a cubic function?

A.  

B.  

C.  

D.  

2) To estimate the length of time that snow fell on this particular day, a student should determine the

A. y-intercept
B. x-coordinate of the vertex
C. y-coordinate of the vertex
D. difference between the x-intercepts
3) What are the characteristics of the graph?

<table>
<thead>
<tr>
<th>Sign of Leading Coefficient</th>
<th>Degree</th>
<th>Number of x-intercepts</th>
</tr>
</thead>
<tbody>
<tr>
<td>A. Positive</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>B. Positive</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>C. Negative</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>D. Negative</td>
<td>3</td>
<td>3</td>
</tr>
</tbody>
</table>

4) The statements that are true are
   A. I and II only
   B. III and IV only
   C. I, II, and IV
   D. I, III, and IV
Use the following information to answer the next question.

A hockey arena seats 1600 people. The cost of a ticket is $10. At this price, every ticket is sold. To obtain more revenue, the arena management plans to increase the ticket price. A survey was conducted to estimate the potential revenue for different ticket prices, as shown below.

<table>
<thead>
<tr>
<th>Ticket price ($)</th>
<th>Potential Revenue ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>16 000</td>
</tr>
<tr>
<td>15</td>
<td>19 500</td>
</tr>
<tr>
<td>20</td>
<td>20 300</td>
</tr>
<tr>
<td>25</td>
<td>14 750</td>
</tr>
<tr>
<td>30</td>
<td>5 500</td>
</tr>
</tbody>
</table>

The data above can be modelled by a quadratic regression function of the form

\[ y = ax^2 + bx + c \]

where \( x \) is the ticket price, in dollars, and \( y \) is the potential revenue, in dollars.

**Numerical Response 1:** The ticket price that would maximize the revenue is $17.2.

Use the following information to answer the next question:

A rain gutter is made from sheets of aluminium that are 28cm wide. The first step is forming the rain gutter is to turn the edges up to form right angles.

The cross-sectional area formed by the turned-up edges affects the water flow. The cross-sectional area, \( A \), can be modelled by the function

\[ A = x(28 - 2x) \]

Where \( x \) is the height of the turned up edges

**Numerical Response 2:**

To the nearest centimeter, the height of the turned-up edge, \( x \), that will maximize the cross-sectional area is 7 cm.
5) Determine which statement is false.
   A. The maximum number of x-intercepts the graph may have is equal to the degree of the function
   B. The maximum number of turning points a graph may have is equal to one more than the degree of the function
   C. The degree and leading coefficient of the equation of a polynomial function indicate the end behavior of the graph of the function
   D. The constant term in the equation of a polynomial function is the y-intercept of its graph.

6) Kyle described the characteristics of \( f(x) = -3x^3 + 5x^2 + 11x \), but there is an error. Identify the error.
   A. This is a polynomial function of degree 3, so I know that the graph will have two turning points
   B. The leading coefficient of this cubic function is negative, so I know that the graph extends from QII to QIV
   C. There is no constant term, so there is no y-intercept.
   D. The domain is \( x \in \mathbb{R} \)

---

Use the following information to answer the next question

A juice box measures 5.0 cm \( \times \) 4.0 cm \( \times \) 12.0 cm and contains 240 mL of juice. The manufacturer wants to design a larger box by increasing each dimension of the juice box by the same amount.

The volume of the larger box can be modelled by the function

\[
V = (5 + x)(4 + x)(12 + x)
\]

where \( V \) represents the volume, in mL, and \( x \) represents the increase in the length of each dimension, in cm.

**Note:** 1 cm\(^3\) = 1 mL

---

**Numerical Response 3:**

If the larger box must hold a maximum of 1 000 mL of juice, the amount, \( x \), by which each dimension of the juice box must be increased, to the nearest tenth of a centimetre, is 2.5 cm.
7) The growth of a tree can be modeled by the function \( h(t) = 2.3t + 0.45 \), where \( h \) represents the height in meters and \( t \) represents the time in years. Approximately how tall will the tree be in 8 years?

A. 18.85 m  
B. 17.15 m  
C. 19.55 m  
D. 16.75 m

8) The average retail price of gas in Canada from 1979 to 2008 can be modeled by the function \( P(y) = 0.008y^3 - 0.307y^2 + 4.830y + 25.720 \), where \( P \) is the price of gas in cents per liter and \( y \) is the number of years after 1979. During which year did the price of gas reach 50¢/L?

A. 1985  
B. 1988  
C. 1991  
D. 1993

9) The polynomial function of \( j(x) = x^2 - 2x - 2 \) is represented by the graph:

A. I  
B. II  
C. V  
D. VI

10) The polynomial function of \( g(x) = -x^3 + 4x^2 - 2x - 2 \) is represented by the graph:

A. III  
B. IV  
C. V  
D. VI
**SINUSOIDAL FUNCTIONS**

**Specific Outcome 8**
Represent data, using sinusoidal functions, to solve problems.

**EXAMPLE 1**
Determine the range, amplitude, period, and equation of the midline for

\[ y = 2 \sin(x + 60) - 2 \]

- \[ \text{range } -2 \leq y \leq 2 \]
- \[ -2 \leq y \leq 2 \]
- \[ \text{amp } = 2 \]
- \[ \text{period } = \frac{2\pi}{1} \]

**EXAMPLE 2**
Ashley boards the Ferris wheel at the Pacific National Exhibition. When the ride begins, her position can be modelled by the function

\[ y = 43 \sin 3.5(x - 0.9) + 47 \]

Where \( y \) represents the height in feet and \( x \) represents the time in minutes.

a) Determine the diameter of the Ferris wheel.

\[ 2(43) = 86 \]

b) How long does it take for the Ferris wheel to complete one revolution?

\[ \text{Period} = \frac{2\pi}{3.5} = 1.795 \text{ minutes} \]

c) How high above the ground is Ashley at the lowest point?

\[ 47 - 43 = 4 \text{ m} \]
EXAMPLE 3

The Bay of Fundy, in the Maritimes, has the highest tides in the world. The height of the water, in metres above the seabed, is shown for one point over 36 h.

<table>
<thead>
<tr>
<th>Hour</th>
<th>Height (m)</th>
<th>Hour</th>
<th>Height (m)</th>
<th>Hour</th>
<th>Height (m)</th>
<th>Hour</th>
<th>Height (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3.5</td>
<td>10</td>
<td>7.0</td>
<td>19</td>
<td>4.6</td>
<td>28</td>
<td>1.6</td>
</tr>
<tr>
<td>2</td>
<td>2.4</td>
<td>11</td>
<td>6.4</td>
<td>20</td>
<td>5.9</td>
<td>29</td>
<td>1.9</td>
</tr>
<tr>
<td>3</td>
<td>1.8</td>
<td>12</td>
<td>5.3</td>
<td>21</td>
<td>6.9</td>
<td>30</td>
<td>2.7</td>
</tr>
<tr>
<td>4</td>
<td>1.9</td>
<td>13</td>
<td>4.0</td>
<td>22</td>
<td>7.2</td>
<td>31</td>
<td>4.0</td>
</tr>
<tr>
<td>5</td>
<td>2.7</td>
<td>14</td>
<td>2.7</td>
<td>23</td>
<td>6.8</td>
<td>32</td>
<td>5.4</td>
</tr>
<tr>
<td>6</td>
<td>3.8</td>
<td>15</td>
<td>1.8</td>
<td>24</td>
<td>5.9</td>
<td>33</td>
<td>6.5</td>
</tr>
<tr>
<td>7</td>
<td>5.2</td>
<td>16</td>
<td>1.7</td>
<td>25</td>
<td>4.6</td>
<td>34</td>
<td>7.1</td>
</tr>
<tr>
<td>8</td>
<td>6.3</td>
<td>17</td>
<td>2.2</td>
<td>26</td>
<td>3.2</td>
<td>35</td>
<td>7.0</td>
</tr>
<tr>
<td>9</td>
<td>7.0</td>
<td>18</td>
<td>3.3</td>
<td>27</td>
<td>2.1</td>
<td>36</td>
<td>6.3</td>
</tr>
</tbody>
</table>

a) Determine the equation of a sinusoidal regression function that models the height of the water.

\[ y = 2.73 \sin\left(0.506x + 3.02\right) + 4.4261 \]

b) How high is the water at high tide, to the nearest tenth of a metre? How high is the water at low tide?

\[ \text{max} = 7.15 \quad (c_1 + a) \]
\[ \text{min} = 1.69 \quad (c_1 - a) \]

c) How long, to the nearest minute, does it take for the tide to cycle from high tide to low tide and back again?

\[ \frac{2\pi}{0.506} = 12.42 \text{ hours} \]

d) Simon plans to go fishing at hour 50. How high, to the nearest tenth of a metre, will the tide be when he begins fishing?

\[ y = 4.031 \text{ m} \]
1) Which of the following graphs best models the height of the point H on the bicycle tire as the bike rolls forward?

A. 

\[
\begin{array}{c}
\text{Height} \\
\text{Time}
\end{array}
\]

B. 

\[
\begin{array}{c}
\text{Height} \\
\text{Time}
\end{array}
\]

C. 

\[
\begin{array}{c}
\text{Height} \\
\text{Time}
\end{array}
\]

D. 

\[
\begin{array}{c}
\text{Height} \\
\text{Time}
\end{array}
\]
Use the following information to answer the next question

A typical wind turbine has blades that are 30 m long set on a tower which is 80 m high.

An equation which represents the height, \( h \), of the top of one of the blades as a function of time, \( t \), in seconds, is given by \( h = 30 \sin(1.5707t) + 80 \)

2) Determine the amplitude and maximum value of this sinusoidal function.

<table>
<thead>
<tr>
<th>Amplitude</th>
<th>Maximum Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>A. 30 m</td>
<td>80 m</td>
</tr>
<tr>
<td>B. 30 m</td>
<td>110 m</td>
</tr>
<tr>
<td>C. 80 m</td>
<td>110 m</td>
</tr>
<tr>
<td>D. 110 m</td>
<td>80 m</td>
</tr>
</tbody>
</table>

3) What are the characteristics of the function \( y = 3\sin\left(\frac{1}{2}x\right) \)?

<table>
<thead>
<tr>
<th>Amplitude</th>
<th>Mid-line</th>
<th>Period</th>
</tr>
</thead>
<tbody>
<tr>
<td>A. 3</td>
<td>0</td>
<td>4\pi</td>
</tr>
<tr>
<td>B. ( \frac{1}{2} )</td>
<td>3</td>
<td>2\pi</td>
</tr>
<tr>
<td>C. 0</td>
<td>( \frac{1}{2} )</td>
<td>3</td>
</tr>
<tr>
<td>D. 3</td>
<td>0</td>
<td>( \frac{1}{2} )</td>
</tr>
</tbody>
</table>
Use the following information to answer the next question

The height of a rider on a Ferris wheel can be modelled by the sinusoidal regression function

\[ h = 6 \sin(1.05t - 1.57) + 8 \]

where \( h \) is the height of the rider above the ground, in metres, and \( t \) is the time in minutes after the ride starts.

4) According to the sinusoidal regression function, the maximum height of the rider above the ground is
   A. 2 m  
   B. 6 m  
   C. 8 m  
   D. 14 m  

**Numerical Response**

When the rider is at least 11.5 m above the ground, she can see the rodeo grounds. During each rotation of the Ferris wheel, the length of time that the rider can see the rodeo grounds, to the nearest tenth of a minute, is \( \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \) min.

Use the following information to answer the next question

A family is going to take a ride on the London Eye, which is a Ferris wheel that continually rotates. Once the family is in a capsule on the ride, the height of the capsule above the ground, \( y \), in metres, can be modelled by the sinusoidal function.

\[ y = a \sin(bx + c) + d \]

Where \( x \) is the time, in seconds, after the family gets in the capsule.

5) In this context, the y-intercept represents the
   A. Length of time it takes for the family to reach the median height of the Ferris wheel
   B. Length of time it takes for the Ferris wheel to complete one revolution
   C. Height of the family at the highest point above the ground
   D. Height of the family at the moment they get in the capsule

Use the following information to answer the next question

The height of the tide, in meters, at a particular harbour can be modelled by the sinusoidal function

\[ h = 4 \sin(0.51t + 1.57) + 6 \]

where \( t \) represents the number of hours after midnight on a particular day.

6) To the nearest minute, the earliest time after midnight on that day that the height of the is at its minimum depth is
   A. 09:14  
   B. 06:10  
   C. 03:05  
   D. 00:09
Matt’s blood pressure is recorded every 0.2 seconds.

<table>
<thead>
<tr>
<th>Time (seconds)</th>
<th>Blood Pressure (mm of Hg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>108</td>
</tr>
<tr>
<td>0.2</td>
<td>122</td>
</tr>
<tr>
<td>0.4</td>
<td>86</td>
</tr>
<tr>
<td>0.6</td>
<td>107</td>
</tr>
<tr>
<td>0.8</td>
<td>123</td>
</tr>
<tr>
<td>1.0</td>
<td>86</td>
</tr>
<tr>
<td>1.2</td>
<td>106</td>
</tr>
</tbody>
</table>

After collecting the data, plotting points and finding the regression equation, Matt decides to research blood pressure on the Internet. He learns that:
- **Systolic** refers to the highest point of blood pressure.
- **Diastolic** refers to the lowest point of blood pressure.

He also finds a chart that categorizes people by their blood pressure.

<table>
<thead>
<tr>
<th>Rating</th>
<th>Systolic</th>
<th>Diastolic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Optimal</td>
<td>&lt; 120</td>
<td>&lt; 80</td>
</tr>
<tr>
<td>Normal</td>
<td>&lt; 130</td>
<td>&lt; 85</td>
</tr>
<tr>
<td>High Normal</td>
<td>130–139</td>
<td>85–89</td>
</tr>
<tr>
<td>Hypertension Stage 1</td>
<td>140–159</td>
<td>90–99</td>
</tr>
<tr>
<td>Hypertension Stage 2</td>
<td>160–179</td>
<td>100–109</td>
</tr>
<tr>
<td>Hypertension Stage 3</td>
<td>&gt; 179</td>
<td>&gt; 109</td>
</tr>
</tbody>
</table>

7) What category does Matt fit into?
A. Optimal
B. Normal
C. High Normal
D. Hypertension (Stage 1)
8) If the minimum height of a gondola on the Ferris wheel is 2 feet, what would be a possible equation representing the function for a gondola's height, $h$, as a function of time, $t$, in minutes?

A. $h(t) = 131\sin 0.314t + 133$
B. $h(t) = 131\sin 0.628t + 133$
C. $h(t) = 131\sin 20t + 264$
D. $h(t) = 134 \sin 20t + 262$